ORBITAL CHARACTERISTICS OF METEOROIDS

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ABSTRACT

The bulk of meteoroidal particles follow pseudo-random orbits and are termed sporadic meteoroids. These are thought to be derived from the correlated streams of particles released by comets, although the mechanisms by which their orbits are dispersed have been the subject of some confusion. By developing techniques to compute the frequency of close encounters with each of the planets, and also the gross outcome of such events, it is shown that most sporadic orbits are a result of gravitational scattering by the giant planets. Jupiter plays the major role.

Although catastrophic impacts with smaller particles limit the lifetimes of meteoroids, this mechanism is not responsible for the bulk of the stream disruption. With a simple model of the zodiacal cloud, the method is also used to find the collisional lifetime of meteoroids including for the first time the dependence upon inclination.

The rate of meteoroid depletion by planetary collisions and hyperbolic ejections resulting from close approaches is calculated. It is found that for Jupiter-crossing meteoroids these losses are as rapid as those due to the Poynting-Robertson effect.

This theory is also applied to six peculiar asteroids, including Hidalgo and Chiron. These prove to have extremely short-lived orbits: large orbital variations occur on a time-scale of only ~10^3 years. It is also shown that Pluto exists in its Neptune-crossing orbit solely because of the stable resonance which prohibits approaches between the two in the present epoch.

The collision rate between the Apollo-Amor-Aten asteroids and each of the terrestrial planets is calculated using all 76 known objects. The result using this new procedure (4-6 Earth impacts per million years) is somewhat higher than previous estimates, indicating that these asteroids do not represent a steady-state population.

CHAPTER 1

INTRODUCTION

In this thesis I investigate the role of close planetary encounters in the evolution of the smaller bodies in the solar system: the main concern is the dispersion of meteoroid orbits, but the theory can equally well be applied to asteroids and comets.

In recent years as a result of spacecraft observations much attention has been focussed upon the dynamics of the cloud of particles responsible for the zodiacal light, the majority being particles smaller than 100 μm. A variety of forces influence the distribution of these particles. Burns et al. (1979) closed their review of the radiative forces at work by noting that new approaches are necessary for evaluating the long-term effects of planetary perturbations and stochastic collisions. This is especially true for larger interplanetary particles, upon which the radiative forces are much less important: such a new approach is developed here.

In particular I intend to deal with those particles which give rise to meteors (observed by radar, photographic, TV, visual or other techniques) when they enter the Earth's atmosphere. These bodies, which can conveniently be considered to range in radius from 100 μm to 1 cm, will be denoted as *meteoroids* although the semantically-incorrect term *meteor* is sometimes more easily used. By convention smaller particles are often called *dust* and much larger bodies *boulders*, *comets* and *asteroids*.

There are many outstanding problems in what might be called the *ecology* of interplanetary particles which have been discussed in detail in several recent books or collections of papers (e.g. Elsässer and Fechtig, 1976; Delsemme, 1977; McDonnell, 1978; Halliday and McIntosh, 1979; and specific references in this text). It is believed that the majority of meteoroids are particles released by comets whilst passing through the inner solar system, and that these replenish the zodiacal light cloud when fragmented in catastrophic collisions. Nevertheless there are major problems of understanding in each of these two steps and much observational and modelling work still remains to be done. The mass distribution index for different populations is an important indicator as to the processes which are occurring: this is reviewed by Hughes (1978) but is not touched upon here.

After release a meteoroid follows an orbit similar to its parent comet since their relative velocity is small. This results in a stream of particles, and hence the meteor showers seen on the Earth at various times of the year. However only ~25% of all meteors are associated with streams, the rest comprising a pseudo-random influx of *sporadic* meteors. The mechanism whereby meteoroids are thrown from their original streams into dissimilar sporadic orbits is another major problem in meteoroid ecology, as was explicitly stated by Dohnanyi (1970; p.3485). Important questions still to be answered for meteoroids are the same as those which were posed by Misconi and Weinberg (1978) in their consideration of the zodiacal dust cloud: "Does Jupiter have the dominant role in changing the orbital elements of the dust, compared to the other planets? What is the magnitude of these perturbations and what are the associated time-scales? How far is the gravitational sphere of influence of each of the planets? Does Mercury affect the dust distribution despite its small mass?" These are the questions which I have attempted to answer in this thesis.

The confusion in this area is well-illustrated by the fact that, in two chapters in the same volume, Dohnanyi (1978) ascribes sporadic production to the scattering of stream meteoroids by the planets (as was first suggested by Plavec in 1956) whilst Hughes (1978) states that sporadics are largely the result of collisions between stream meteoroids and zodiacal cloud particles. Although it has been known since the work of Zook and Berg in 1975 that the eventual fate of a meteoroid is most likely a catastrophic impact, and also the high mass index of sporadics shows that collisions must play some part, it is by no means clear that the majority of sporadics originate in meteoroid-dust collisions. To date there has been no quantitative footing to ideas upon the providence of sporadic meteor orbits, which are totally different to those of comet-associated streams (e.g. Hawkins, 1962).

A number of different approaches have been used to investigate the orbital characteristics of the smaller bodies in the solar system. Concentrating upon the origin of meteorites, Arnold (1964, 1965) used an encounter probability method coupled with a Monte Carlo simulation of the outcome of close encounters. Other Monte Carlo simulations, or variants, have been used in studies of the origin of asteroids and comets (Everhart, 1968; 1969; 1972; 1973; Carusi and Pozzi, 1978a; 1978b; Carusi and Valsecchi, 1980a;

1980b; Rickman and Froeschle, 1980), although these require vast amounts of computing time. Kresak (1982) has discussed the use of the Tisserand invariant (Jacobi constant) and the D-criterion (described in chapter 4) in differentiating between cometary, asteroidal and meteoroidal orbits. Alfven and Arrhenius (1976) have put forward many novel ideas upon the interrelation of meteoroids and comets, and have discussed the origin of sporadic orbits in general terms. Despite the above advances there has been no definitive answer to the origin of sporadic orbits since Plavec (1956) suggested the dispersion of meteor streams by planetary close encounters.

Recently some work has been done upon exact calculations of close encounters between specific streams and Jupiter (Carusi et al. 1981; 1982a; 1982b; 1983). The results show that the stream is destroyed by the encounter, and the stream particles are dispersed into the sporadic background. However, this is for a particular case only, where the meteoroid trajectories are chosen so as to intersect the planet. To find the gross effect of planetary encounters the probability (or frequency) of such encounters is required, and also some measure of the outcome of these events: these factors I evaluate as follows.

The first step is to deduce the frequency of meetings between a particle in an arbitrary orbit and each of the planets. The technique used to accomplish this, detailed in chapter 2, is due to Kessler (1981). This method was preferred, rather than the limited methods based upon the work of Öpik (1951; 1976) which have been in favour to date, because the relative velocity and encounter geometry of particle and planet is immediately available in all encounter situations. These parameters are needed in the second step, described in chapter 3, which involves calculating the partition between the different outcomes of such an encounter (i.e. collision with the planet, ejection from the solar system, or a perturbation of the original elliptical orbit). A computer program which performs these calculations is included as Appendix 1.

Use of this program to find the sporadic production rate is discussed in chapter 4; the time-scale for sporadic production must be compared to the time-scales for the other dynamical effects which are also reviewed there. It is confirmed that the limiting lifetime for meteoroids is defined by impacts with zodiacal cloud particles. The techniques of chapter 2 are also used to make a new deduction of the meteor collisional lifetime using a simple model of the zodiacal cloud based upon spacecraft measurements, the results confirming previous rudimentary estimates. This is the first time that the dependence of the inclination of the meteoroid orbit has been included in the calculation (c.f. Leinert et al, 1983) and is therefore a significant improvement upon previous methods.

Many of the factors influencing the lifetimes discussed in chapter 4 are dependent upon the meteoroid density. There is now a wealth of evidence that whilst the smaller particles (e.g. 1-100 μm dust, the dynamics of which were investigated by Kresak, 1976) have densities of 3-4 gm cm^{-3} (Hanner, 1980), meteoroids have a much lower density in line with a looser structure (Hughes, 1978). The densities measured by radar techniques show a large scatter but a value of ~0.8 gm cm^{-3} seems appropriate (Verniani, 1973). Mukai and Fechtig (1983) have put forward a theory to explain how gradual closer packing of a meteoroid can slowly increase its density. In view of the above, and for the sake of simplicity, I shall adopt a mean density of 1 gm cm^{-3} throughout.

Attention is next turned to meteoroid losses caused by the planets (planetary impact or ejection on an hyperbolic orbit), and sporadic meteoroid production in close encounters. The results for a wide variety of sample orbits are calculated in chapter 5, and in chapter 6 the important time-scales are calculated for each distinct set of orbital elements. A number of new and important deductions are made, the most significant being:

a) Meteoroids in Jupiter-crossing orbits suffer losses due to planetary collisions and (more abundant) ejections at a rate comparable to depletion by the Poynting-Robertson effect;

b) Jupiter-crossing stream meteoroids are dispersed into sporadic orbits by close encounters much faster than they are lost from the meteoric complex.

As aforementioned, the theory developed here can also be applied to larger bodies. In chapter 7 I investigate the orbital evolution of several objects of particular interest: the two giant asteroids Hidalgo and Chiron, each of which crosses two of the giant planets, and four smaller asteroids which cross Jupiter. All six are shown to have lifetimes in their present orbits which are extremely brief compared to the age of the solar system. Only one planet (Pluto) crosses the orbit of another, and its case is discussed in some detail.

The meteoroid orbits used in chapter 5 are typical of a variety of comets. For the sake of brevity I do not additionally consider any specific comets, although it is noteworthy that the evolution of cometary orbits can also be researched using these techniques. In particular, the principle of reversibility implies that the ejection probabilities calculated for the closed orbits used here are upper limits for the capture probabilities of hyperbolic comets entering the solar system. The amount of computation is modest compared to the Monte Carlo simulations of comet capture carried out by Everhart (1969; 1973), and others.

There is one other category of interplanetary particle which is of much interest: the Apollo-Amor-Aten asteroids, which cross the terrestrial planets. These are removed quickly due to planetary impacts, and the necessary supply has been the subject of intense study over the past decade. In Appendix 2, consisting of a substantial paper which has been accepted for publication, I re-evaluate the collisional lifetimes of these objects using the new techniques embodied by this thesis. The results allow a prediction of the cratering rate on Mercury, Venus, the Earth and Mars in the present epoch. Comparison with the long-term lunar and terrestrial crater record adds weight to the hypothesis that the present Apollo-Amor-Aten population is not in a steady-state, but is the remnant of a wave of comets which entered the planetary region in astronomically-recent time.

CHAPTER 2

THE PROBABILITY OF AN ENCOUNTER BETWEEN TWO OBJECTS IN ARBITRARY KEPLERIAN ORBITS

2.1 INTRODUCTION

In this chapter I derive the probability of an encounter between two arbitrary orbiting objects, expanding upon the description of Kessler (1981). This method is easily implemented by computer. Distinct methods have been those of Öpik (1951), Wetherill (1967), and Shoemaker et al. (1979).

Öpik (1951) developed an approximate theory for calculating the probability of an impact by a minor body in an elliptic orbit upon a planet in an orbit taken to be circular. This was applied by Öpik to a number of particular objects in the solar system (Öpik, 1963, 1966a; and numerous other papers). The limitations imposed by the approximations made have been discussed (Öpik, 1976).

Wetherill (1967) derived a more general set of equations by allowing both objects to be in non-circular orbits, thus deriving a more complex but more precise formalism which he applied to the problem of collisions in the asteroid belt.

Shoemaker et al. (1979) used a similar but alternate derivation to that of Wetherill in order to estimate the asteroidal collision rate with the Earth. Their method is useful in that it incorporates the secular perturbations which alter the orbits of the objects in question, whereas Kessler's method utilizes only the osculating orbital elements.

Thus Kessler's equations give a zero impact probability with the Earth for an Amor asteroid, since it is not presently an Earth-crosser, whereas secular variations may change it into an Earth-crossing (Apollo) asteroid eventually, with a finite collision probability. However, an exact solution to the equations of Shoemaker et al. (1979) is not yet possible, severely limiting their application.

Recently Greenberg (1982) has derived a new geometrical formalism for orbital encounters avoiding many of the approximations of Öpik and Wetherill. With some refinement it may be possible to extend Greenberg's method to include the effects of secular perturbations, and to find the gross effects of close encounters. (e.g. to calculate the probability of ejection from the solar system by a different technique to that described in chapter 3).

In this chapter it is assumed that the type of event in question is an actual impact, so that the collision cross-section is the cross-section of interest (equation 2). In chapter 3 the cross-sections for deflection, ejection, and collision, are all derived. Thus to find the probability of an ejection or deflection it is simply necessary to replace $\bar{\sigma}_j$ in equation 7 by the relevant cross-section.

Kessler's method is analogous to the kinetic theory of gases. The orbit of each object is described by semi-major axis a, eccentricity e, and inclination i. An essential feature of the method is that the argument of periapsis is taken to be random: Öpik (1951) pointed out that this assumption is valid over astronomical time-bases due to secular perturbations and precession. This assumption can also be justified cyclically. Results using Kessler's method, as presented in the later chapters of this thesis, show that close encounters producing deflections of a few degrees occur for planet-crossing objects on time-scales of 10^6 years or less. Smaller deviations will be very much more frequent, and time-scales of 10^4-10^5 or less years are applicable to minor changes in a particle's orbit due to planetary perturbations which cause an appreciable alteration of the argument of periapsis. The argument of periapsis (or the longitude of periapsis and the longitude of the node) is hence essentially random, and Kessler's method is valid. An exception to this would be a particle whose orbit was 'protected' by being in some form of resonance with a planet (e.g. the Trojan or the Hilda asteroids). Such a resonance is generally possible if the minor body crosses one planet only. For example, the resonance which prevents Toro from striking the Earth is not stable since this asteroid crosses Mars also (Shoemaker et al., 1979).

The collision probability for two secondary bodies orbiting a given primary object is a function of the two sets of orbital elements (a,e,i), and the physical characteristics of these bodies (mass and radius, m and r). Therefore, with M the mass of the primary, the collision probability over a long time-base with random argument of periapsis is:

$$P = P(M;\ a_1,\ e_1,\ i_1,\ m_1,\ r_1;\ a_2,\ e_2,\ i_2,\ m_2,\ r_2)$$

This technique is especially valuable in dealing with objects in heliocentric orbits (M the solar mass, but it can be used for any Keplerian orbits, such as stars about the centre of their galaxy, or moons orbiting their parent planet. The method was developed to find the collisional lifetime of artificial satellites in Earth-orbit by Kessler and Cour-Palais (1978), and was applied by Kessler (1981) to the outer moons of Jupiter

2.2 BASIS OF THE METHOD

Take the volume of space in which it is possible for the two objects to collide to be split up into small volume elements ΔU, each of which is small compared to the uncertainty in the orbital paths of the objects. Then the positions of each object within any volume element is essentially random, as are the molecules in a gas, and the flux of one object is

$$F = Sv \tag{1}$$

where v is the velocity of the object, and S its mean 'spatial density' within the volume ΔU; that is, inside of this volume element there are on average S bodies having this orbit, per unit volume.

The number of impacts upon an area A in time t is then given simply by FAt . However, in general the objects have an appreciable gravitational field so that gravitational focussing increases the effective collisional cross-section to become (Öpik, 1951):

$$\sigma = \pi(r_1 + r_2)^2 \left[1 + \frac{2G(m_1 + m_2)}{(r_1+r_2)V^2}\right]^{\frac{1}{2}} \qquad (2)$$

In the case of a minor body such as a comet or asteroid striking a planet, r_2 and m_2 are small. Then the term outside of the bracket is just the geometrical cross-section of the planet, and the inside of the bracket is unity plus the ratio of the square of the planetary escape velocity to the square of the encounter velocity, V.

The number of collisions in time t is then

$$N = F\sigma t \qquad (3)$$

so that the collision rate is, from (1) and (3),

$$\frac{N}{t} = Sv\sigma \qquad (4)$$

However, this would assume that one body is held stationary in the volume element; in fact there is only a small chance that both objects will be in the volume element at the same time, so that (4) becomes

$$\frac{N}{t} = S_1 S_2 V\sigma\Delta U \qquad (5)$$

where S_1 and S_2 are the respective spatial densities of the two objects in this particular volume element ΔU, σ is the mutual collision cross-section given by (2), and V is the relative velocity of the two at this position.

The total collision probability between the two is the sum of the collision rates in all volume elements accessible to both objects:

$$P = \int_{\text{volume}} S_1 S_2 V\sigma \, dU \tag{6}$$

This assumes that the collision probability is sufficiently small such that many orbits are executed. Equation (6) can be solved numerically by defining volume elements of such a size that the spatial density does not vary appreciably within the element; then

$$P = \sum_j \overline{S_{1j}}\ \overline{S_{2j}}\ \overline{V_j}\ \overline{\sigma_j}\ \Delta U_j \tag{7}$$

where the bars indicate that the mean value throughout the element is used.

Once the mean encounter velocity is known (section 2.4), the collision cross-section can be calculated from equation (2). In section 2.3 the expression for the spatial density is deduced.

2.3 SPATIAL DENSITY

If the argument of periapsis is random, then the spatial density is a function of distance to primary (R) and ecliptic latitude (β) only, so that

$$S = S(R,\beta)$$

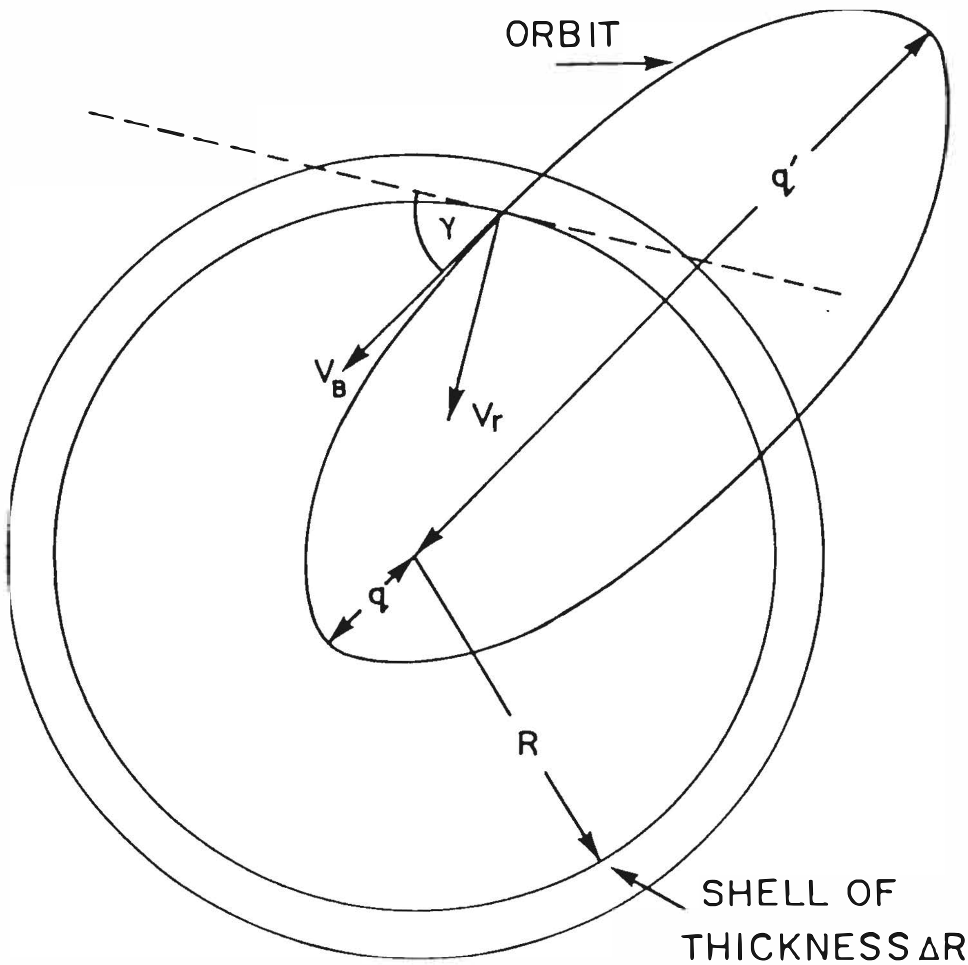


FIGURE 1

The variation of spatial density with distance from the primary.

In addition, S is separately dependent upon these two parameters:

$$S(R,\beta) = Q(R)B(\beta) \tag{8}$$

Here Q is the spatial density at R averaged over all latitudes, and B is the ratio of the spatial density at β to the spatial density averaged over all latitudes. Q depends upon the size and shape of the orbit, described by a and e; or equally well by periapsis distance q and apoapsis distance q':

$$Q(r) = Q(a,e) = Q(q,q')$$

B depends only upon the inclination:

$$B(\beta) = B(i)$$

and the two functions Q and B are derived separately.

2.3.1 Radial Spatial Density Variation

Consider a spherical shell of thickness ΔR, at a distance R from the primary such that $(q \leqslant R \leqslant q')$, as in Figure 1.

The orbiting body has a velocity V_B with radial component

$$V_r = V_B \sin \gamma \tag{9}$$

V_r is taken to be positive towards the primary so that from apoapsis to periapsis ($0° \leqslant \gamma \leqslant 90°$), and from periapsis to apoapsis ($270° \leqslant \gamma \leqslant 0°$). The shell has an internal distance R, external distance R', mean distance

$$\bar{R} = (R + R')/2$$

and a volume

$$\Delta U = 4\pi R^2 \Delta R \tag{10}$$

as long as ($\Delta R << R$).

In each orbit the body passes through the shell twice, so that the time spent within the shell per orbit is

$$\Delta t = 2\Delta R/V_r \tag{11}$$

The velocity of the body is

$$V_B = \left[GM\left(\frac{2}{R} - \frac{1}{a}\right)\right]^{\frac{1}{2}} \tag{12}$$

G being the universal constant of gravitation.

Equations (9) and (12) will allow Δt to be calculated from (11) as long as sin γ is known.

The angular momentum per unit mass is:

at q

$$qV_{Bq} \cos 0° = qV_{Bq} = \left[GM\left(\frac{2}{q} - \frac{1}{a}\right)\right]^{\frac{1}{2}} q \tag{A}$$

at q'

$$q'V_{Bq'}\cos 0° = q'V_{Bq'} = \left[GM\left(\frac{2}{q'} - \frac{1}{a}\right)\right]^{\frac{1}{2}} q' \qquad \text{(B)}$$

at R

$$RV_{BR}\cos\gamma = \left[GM\left(\frac{2}{R} - \frac{1}{a}\right)\right]^{\frac{1}{2}} R\cos\gamma \qquad \text{(C)}$$

Equating (A),(B) and (C) gives

$$2q - q^2/a = 2q' - q'^2/a = \left(\frac{2}{R} - \frac{1}{a}\right) R^2\cos^2\gamma$$

or

$$\cos^2\gamma = \frac{2qa-q^2}{R(2a-R)} = \frac{2q'a-q'^2}{R(2a-R)}$$

However,

$$a = (q+q')/2$$

so that

$$qq' = 2qa - q^2 = 2q'a - q'^2$$

and then

$$\cos^2\gamma = \frac{qq'}{R(2a-R)} \qquad (13)$$

Since $\sin^2\gamma = 1 - \cos^2\gamma$, equations (9), (11) and (12) give:

$$V_r = \left[GM\left(\frac{2}{R} - \frac{1}{a}\right)\left(1 - \frac{qq'}{R(2a-R)}\right)\right]^{\frac{1}{2}} \qquad (14)$$

The spatial density is just

$$Q = \Delta t/T\Delta U \qquad (15)$$

where the period T is

$$T = 2\pi(a^3/GM)^{\frac{1}{2}} \qquad (16)$$

and then (10),(14),(15) and (16) render

$$Q(R) = 1/4\pi R^2 a^{3/2}\left[\left(\frac{2}{R} - \frac{1}{a}\right)(1-qq'/R(2a-R))\right]^{\frac{1}{2}}$$

which, with substitution of $a = (q+q')/2$, gives

$$Q(R) = 1/4\pi^2 Ra[(R-q)(q'-R)]^{\frac{1}{2}} \tag{17}$$

This is the probability of finding the body within a shell of unit volume at distance R, averaged over all latitudes. For $(R < q)$ or $(R > q')$, $Q = 0$. The method for dealing with Q as $(R \to q)$ or $(R \to q')$ is dealt with in section 2.3.5.

2.3.2 Latitudinal Spatial Density Variation

The orbiting body is now defined to be within a shell of thickness ΔR at distance R. The requirement is to find the probability of finding the body within a band of latitude from β to $\beta + \Delta\beta$.

Referring to Figure 2, XY is the ecliptic plane. The body is imagined to be held stationary at the point O, given by (R,β), and is then taken around a circular orbit of inclination i (the same as the real orbit) so that it traverses path WOX. All possible values for the argument of periapsis of the real orbit are scanned in this way.

If the argument of perihelion moves with an angular velocity ω, which is constant since the imaginary path is

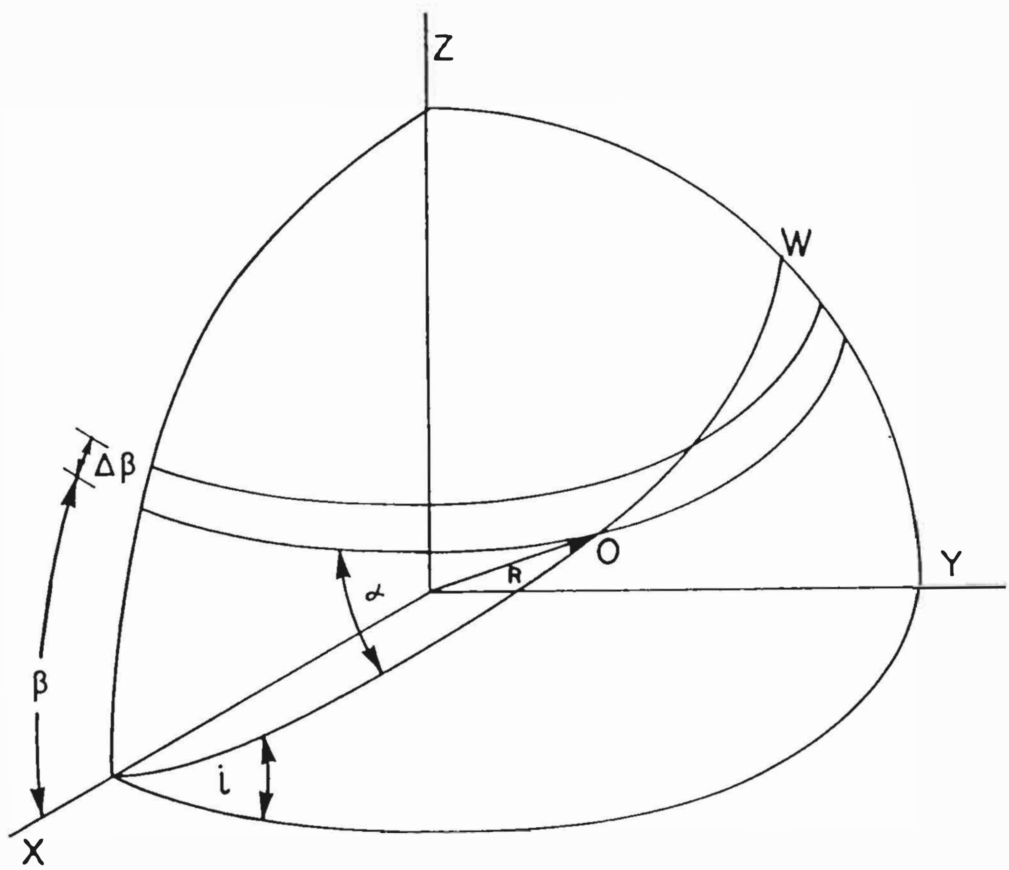


FIGURE 2

The variation of spatial density with ecliptic latitude.

circular, then the period of this imaginary orbit is

$$T' = 2\pi/\omega \tag{18}$$

The time taken to cross the latitude band of width $\Delta\beta$ is just $(\Delta\beta/\omega\sin\alpha)$ where α is the angle which the path makes with the line of constant latitude. Since the object crosses this latitude twice in every orbit, the total time spent in this band per orbit is

$$\Delta t' = 2\Delta\beta/\omega\sin\alpha \tag{19}$$

Equations (18) and (19) therefore give the fraction of the object's time spent between β and $\beta+\Delta\beta$ as

$$\frac{\Delta t'}{T'} = \frac{\Delta\beta}{\pi\sin\alpha} \tag{20}$$

In a similar way to (15), since we need to consider a unit volume given by equation (10),

$$\frac{B}{4\pi R^2\Delta R} = \Delta t'/T'\Delta U' \tag{21}$$

and $\Delta U'$ is required in order to calculate B.

The volume of a hemispherical shell is $(2\pi R^2\Delta R)$, and the volume element decreases as the cosine of the latitude. Thus

$$\Delta U' = 2\pi R^2 \cos\beta \ \Delta R\Delta\beta \tag{22}$$

and putting (20) and (22) into (21) renders:

$$\frac{B}{4\pi R^2\Delta R} = 1/2\pi^2 R^2\sin\alpha \ \cos\beta \ \Delta R$$

or

$$B = 2/\pi \sin\alpha \cos\beta \tag{23}$$

which is valid for $0° \leqslant \beta \leqslant i$; $B = 0$ at higher latitudes. Since

$$\cos i = \cos\alpha \cos\beta$$

so that

$$\sin\alpha \cos\beta = [\sin^2 i - \sin^2\beta]^{\frac{1}{2}}$$

and hence

$$B(\beta) = 2/\pi[\sin^2 i - \sin^2\beta]^{\frac{1}{2}} \tag{24}$$

Treatment of this equation as $(\beta \to i)$ is described in section 2.3.6.

2.3.3 Net Spatial Density Variation

Equations (17) and (24) can now be substituted into (8) to give the net spatial density. Since $\bar{R}$ and $\bar{\beta}$ are better measures than R and β, write

$$\bar{S}(R,R',\beta,\beta') = 1/2\pi^3 a\bar{R}[(\sin^2 i-\sin^2\bar{\beta})(\bar{R}-q)(q'-\bar{R})]^{\frac{1}{2}} \tag{25}$$

which is valid for

$$(q \leqslant R,\ R' \leqslant q')$$

and

$$(0° \leqslant |\beta,\beta'| \leqslant i) \quad .$$

The spatial density is zero outside of these limits, but care must be taken as these limits are approached.

2.3.4 Integration Methods

The singularities ocurring when $(R \rightarrow q)$, $(R \rightarrow q')$, or $(\beta \rightarrow i)$ are integrable as follows. The average spatial density within a volume is

$$\bar{S} = \int SdU / \int dU \tag{26}$$

which must be separated into radial and latitudinal terms. For very small volume elements equation (22) is

$$dU = 2\pi R^2 \cos\beta \; dR \; d\beta$$

and thus using (8) equation (26) becomes

$$\bar{S} = \frac{\iint Q(R) B(\beta) R^2 \cos\beta \; dR \; d\beta}{\iint R^2 \cos\beta \; dR d\beta} \tag{27}$$

Since R and β are independent, the integrals are separable:

$$\bar{S}(R,R',\beta,\beta') = \bar{Q}(R,R') \; \bar{B}(\beta,\beta') \tag{28}$$

Here

$$\bar{Q}(R,R') = \int_R^{R'} Q(R) R^2 dR / \int_R^{R'} R^2 dR \tag{29}$$

is the mean spatial density between R and R' averaged over all latitudes, and

$$\bar{B}(\beta,\beta') = \int_{\beta}^{\beta'} B(\beta)\ \cos\beta\ d\beta \Big/ \int_{\beta}^{\beta'} \cos\beta\ d\beta \qquad (30)$$

is the ratio of the mean spatial density between β and β' to the mean spatial density over all latitudes.

Solutions to equations (29) and (30) now are required.

2.3.5 Spatial Density as $(R\to q)$ or $(R\to q')$

Let the thickness of the spherical shell (ΔR) be sufficiently small such that the radial distance at any point can just be set to the average radius:

$$\int_{R}^{R'} R^2 dR \simeq \bar{R}^2 \int_{R}^{R'} dR = \bar{R}^2 \Delta R$$

and

$$\int_{R}^{R'} R^2 Q(R) dR \simeq \bar{R}^2 \int_{R}^{R'} Q(R) dR$$

Thus (29) becomes

$$\bar{Q}(R,R') = \frac{1}{\Delta R} \int_{R}^{R'} S(R) dR$$

and by substituting (17) this is

$$\bar{Q}(R,R') = \frac{1}{4\pi^2 a\bar{R}\Delta R} \int_{R}^{R'} [(R-q)(q'-R)]^{-\frac{1}{2}} dR$$

After evaluation of the integral this becomes

$$\bar{Q}(R,R') = \frac{1}{4\pi^2 a\bar{R}\Delta R}\left[\arcsin\left(\frac{2R'-2a}{q'-q}\right) - \arcsin\left(\frac{2R-2a}{q'-q}\right)\right] \qquad (31)$$

which is valid for all volume elements entirely outside of the periapsis ($R > q$) and inside apoapsis ($R' < q'$).

If ($R \leqslant q$) then the square bracket becomes

$$[\arcsin\left(\frac{2R'-2a}{q'-q}\right) + \frac{\pi}{2}] \quad ;$$

if ($R' \geqslant q'$) then the square bracket becomes

$$\left[\frac{\pi}{2} - \arcsin\left(\frac{2R-2a}{q'-q}\right)\right] \quad ;$$

if ($R > q'$) or ($R' < q$) then

$$\bar{Q}(R,R') = 0.$$

For an extremely low eccentricity orbit it is possible that both ($R \leqslant q$) and ($R' \geqslant q'$), in which case the square bracket is just π, and equation (31) is then

$$\bar{Q}(R,R') = 1/4\pi a\bar{R}\Delta R$$

However, since q and q' are effectively the limits of R and R', and $a = (q+q')/2 = \bar{R}$ here, this becomes

$$\bar{Q}(R \leqslant q, R' \geqslant q') = 1/4\pi\bar{R}^2\Delta R$$

It should be noted that the equivalent to this in Kessler (1981)(equation 11A) contained a typographical error. His equation 10A should also have been for the mean spatial density (Kessler, 1984, personal communication).

A numerical comparison of the results of equations (29) and (31) shows that the approximation used in deriving (31) leads to a typical error of 0.3% for a thousand-point summation in place of the true integral.

2.3.6 Spatial Density as ($\beta \rightarrow i$)

A solution to equation (30) is required. The value of the denominator is simply ($\sin\beta' - \sin\beta$), but the numerator is more complicated. Using equation (24) the numerator is

$$\int_{\beta}^{\beta'} B(\beta)\cos\beta \, d\beta = \int_{\beta}^{\beta'} \frac{2\cos\beta}{\pi[\sin^2 i - \sin^2\beta]^{\frac{1}{2}}} d\beta$$

$$= \frac{2}{\pi \sin i} \int_{\beta}^{\beta'} \frac{\cos\beta}{\left[1 - \left(\frac{\sin\beta}{\sin i}\right)^2\right]^{\frac{1}{2}}} d\beta$$

$$= \frac{2}{\pi} \left[\arcsin\left(\frac{\sin\beta'}{\sin i}\right) - \arcsin\left(\frac{\sin\beta}{\sin i}\right)\right]$$

Thus equation (30) becomes

$$\bar{B}(\beta,\beta') = \frac{2}{\pi(\sin\beta' - \sin\beta)} \left[\arcsin\left(\frac{\sin\beta'}{\sin i}\right) - \arcsin\left(\frac{\sin\beta}{\sin i}\right)\right] \tag{32}$$

This is the average spatial density between latitudes β and β' compared to the spatial density averaged over all latitudes.

It must be remembered that the latitude can take values between -90° and +90°, and that the ratio

$(\sin(\beta \text{ or } \beta')/\sin i)$ cannot exceed unity. Therefore when $(\beta' \geqslant i)$ or $(-\beta \geqslant i)$ the relevant arcsine takes the value $(\pi/2)$. If $(\beta \geqslant i)$ or $(-\beta' \geqslant i)$, $\bar{B}(\beta,\beta') = 0$.

It is easily seen that since $\beta' > \beta$ always, the factor outside of the bracket ensures that the calculated spatial density is positive.

2.3.7 Overall Spatial Density

Equations (31) and (32) can now be substituted into (28) to give the mean spatial density between radial distances R and R' and between latitudes β and β':

$$\bar{S}(R,R',\beta,\beta')$$

$$= \frac{1}{2\pi^3 a\bar{R}\Delta R(\sin\beta' - \sin\beta)} \left[\begin{array}{l} \arcsin\left(\frac{2R'-2a}{q'-q}\right) \\ -\arcsin\left(\frac{2R-2a}{q'-q}\right) \end{array} \right] \left[\begin{array}{l} \arcsin\left(\frac{\sin\beta'}{\sin i}\right) \\ -\arcsin\left(\frac{\sin\beta}{\sin i}\right) \end{array} \right] \tag{33}$$

This formula is necessary when R or R' are close to periapsis or apoapsis; and when the absolute values of the latitudes β or β' are close to the inclination of the orbit. Otherwise equation (25) is used to find the spatial density.

2.4 RELATIVE VELOCITY

The magnitude of the relative velocity of the two bodies in a particular volume element is required. The individual velocities can be calculated by recourse to equation (12), and then the relative velocity from

$$V^2 = V_{B1}^2 + V_{B2}^2 - 2V_{B1}V_{B2} \cos\phi \tag{34}$$

as long as the encounter angle ϕ is known. Figure 3 represents a coordinate system centred upon the point of intersection, with the XY plane perpendicular to the spherical shell in Figure 1. The angle ϕ can be calculated by spherical trigonometry as:

$$\cos \phi = [\sin\gamma_1 \sin\gamma_2] + [\cos\gamma_1 \cos\gamma_2 \cos(\alpha_1-\alpha_2)] \tag{35}$$

The values of $\cos \gamma_1$ and $\cos \gamma_2$ are given by equation (13), taking the positive square root in each case since ($270° \leqslant \gamma \leqslant 90°$). Hence $\sin\gamma_1$ and $\sin\gamma_2$ can be calculated, but these can take either sign: from apoapsis to periapsis ($0° \leqslant \gamma \leqslant 90°$) so that $\sin\gamma$ is positive. From periapsis to apoapsis the converse is true.

In considering Figure 2 it was seen that

$$\cos i = \cos\alpha \cos\beta$$

so that α_1 and α_2 can be calculated from the inclination and the latitude:

$$\cos\alpha = \cos i/\cos\bar{\beta} \tag{36}$$

where

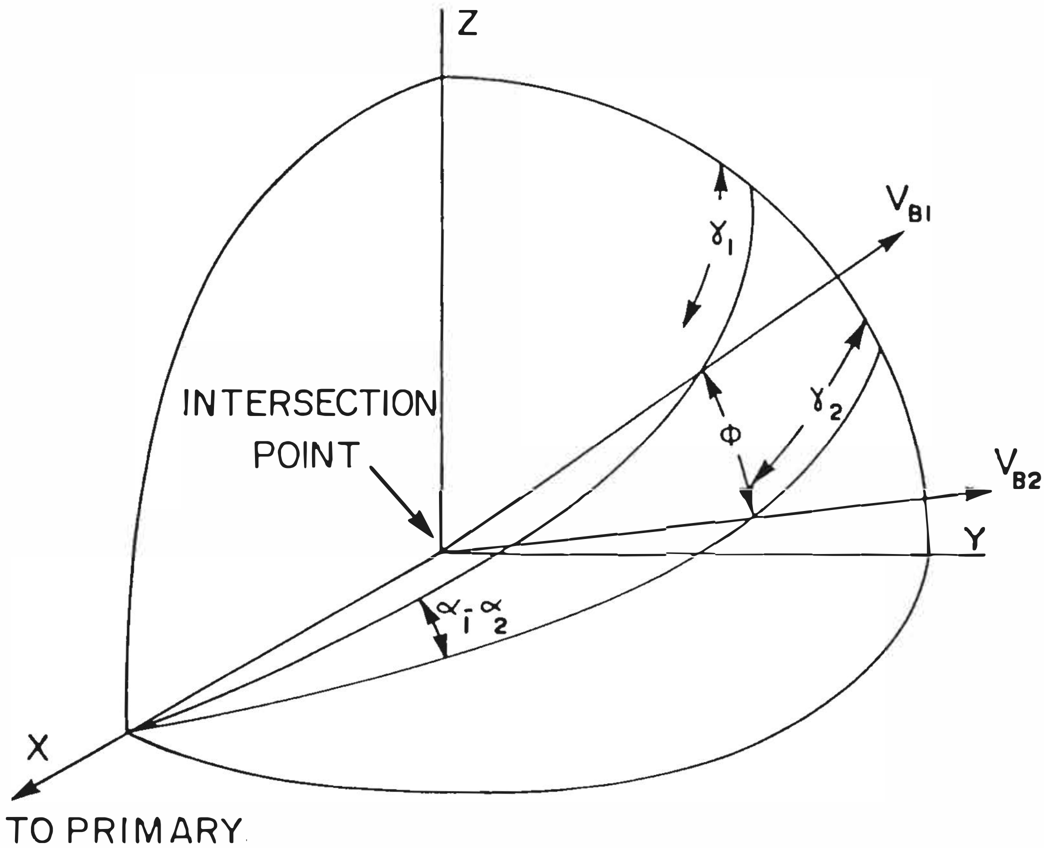


FIGURE 3

Relative velocity of the two colliding objects.

$\bar{\beta} = (\beta+\beta')/2$

Since $270° \leqslant \alpha \leqslant 90°$, $\cos\alpha$ is positive. However, $(\alpha_1-\alpha_2)$ can take any value so that $\cos(\alpha_1-\alpha_2)$ can be positive or negative.

It is seen therefore that each of the square brackets in equation (35) can be positive or negative, and four values for intersection angle ϕ are possible in any particular volume element:

$$\cos\phi = \pm[\sin\gamma_1 \sin\gamma_2] + [\cos\gamma_1 \cos\gamma_2 \cos(\alpha_1 \pm \alpha_2)] \tag{37}$$

Hence there are four possible intersection velocities resulting from equation (34). For the purposes of calculating the mean collision probability via equation (7), the mean encounter velocity is required: this is just the average of the four possible collision velocities. However, as was made clear by Kessler (1981), this is not the same as saying that all collision velocities are equally likely: the higher velocities render a higher likelihood of collision. (An exception to this is when a minor body encounters one of the giant planets: the enhancement of the gravitational cross-section over the geometrical cross-section, given by equation (2), depends upon the reciprocal of the square of the relative velocity. Because of this the probability of a collision at a high velocity, such as when the minor body is in a retrograde orbit, is less than that of a low velocity collision).

2.5 COLLISION PROBABILITY

All terms are now available for the evaluation of equation (7). The two spatial densities $\bar{S}_1$ and $\bar{S}_2$ are given by (25) or (33), bearing in mind the limitations of validity given after equation (25). The mean velocity is found from (34) using (37) and the mean radius vector $\bar{R}$ from (12). The collision cross-section is calculated from (2) using the mean relative velocity. Finally the volume element is given by (22) with the mean parameters being used; i.e.

$$\Delta U = 2\pi \bar{R}^2 \cos \bar{\beta} \; \Delta R \; \Delta\beta \qquad (38)$$

Equation (7) can now be numerically integrated. This must involve a compromise between precision and the computation time involved. However, since an exact collision probability lacks physical usefulness, a precision of better than 1% is not required. In fact the assumptions and approximations made above do not warrant more exact computation. An error limit of 10% is entirely satisfactory.

Kessler (1981) used a radial jump of $\Delta R \leqslant 0.1R$, producing an error of less than 1% in the radial dependence (c.f. section A.3.5). His latitude bands were $\Delta\beta = 3°$. It was found here that 100 iterations in radius ($\Delta R = 0.01$ R at worst) and 50 iterations in latitude ($\Delta\beta = 3\overset{\circ}{.}6$ at worst) is sufficiently precise for all orbits, bearing in mind the desired error limit. It is comparatively simple to program for a self-adjusting number of iterations (i.e.

investigate the volume of space accessible to both bodies and then subdivide this into a suitable number of volume elements by defining ΔR and $\Delta\beta$). The summation of (7) is then over this number of volume elements, j.

A word of warning to future computors: ensure that the volume elements straddle ($R = q, q'$) and ($\beta = 0°, \pm i$).

CHAPTER 3

THE RESULT OF AN ENCOUNTER BETWEEN A MINOR BODY AND A PLANET

3.1 INTRODUCTION

In this chapter I show how an estimate can be made of the result of a close encounter between a body of negligible mass (e.g. a comet, asteroid or meteoroid) and one of the planets. The general method used here is a development of the techniques described by Weidenschilling (1975 a), and also by Fernandez (1978).

The result of a close encounter can be an ejection from the solar system, an orbital disruption, or a collision with the planet. I refer to these respectively as being an ejection, deflection, or collision, and the cross-section for each of these events is required for use in equation (7).

An exact analytical solution for the path taken by the minor body, hereafter termed the 'particle', is not possible; this is the classic 'three-body problem'. A number of alternative approaches have been used, and mostly the encounter is considered as being separable into two distinct two-body phases. Outside of the 'sphere of influence' of the planet (defined later in this chapter) the particle motion is considered to be governed solely by the attraction of the Sun, whilst inside this region the particle's path is taken to be a planetocentric hyperbola. This simplistic assumption is useful since then the particle's velocity relative to the planet is the same at the entry and exit points to the sphere of influence, only its direction is changed. This separation into two, two-body problems has been used extensively by Öpik (1966 b), Bandermann and Wolstencroft (1970, 1971), Weidenschilling (1975 a,b), Cox et al. (1978) and Fernandez (1978). For very low encounter velocities this assumption breaks down (Dole, 1962; Giuli, 1968), but is of minor importance here. As might be expected, exact numerical integrations of the orbits of minor bodies close to a planet with full consideration of the solar field indicate that the approach used here (two distinct two-body problems) does not allow an exact prediction of the outcome of particular encounters (Carusi and Pozzi, 1978 a,b). This is because the mid-range perturbations, outside of the sphere of influence, are not negligible. However my intention is only to find approximate values for the probabilities of collision, deflection and ejection, rather than to plot exact orbital changes, so the simple basis of the treatment is entirely justified.

3.2 DEFLECTION PRODUCING A GRAZING IMPACT

Let a particle of negligible size and mass approach a planet of mass m_1 and radius r . Using the two-body approximation, at a large distance from the planet the particle is taken to travel in a straight line with velocity V relative to the planet (from equation 34). Let the impact parameter (the minimum distance which the particle would pass from the centre of the planet if it continued on a linear path) be b. Let b_g be the impact parameter which actually results in a grazing collision with the planet. This means that b_g is the maximum impact parameter which results in a physical collision, so that πb_g^2 is the collision cross-section. I will follow Weidenschilling (1974) to find the value of b_g and hence the angular deflection producing a grazing impact, χ_g.

The attraction of the planet results in the particle having an accelerated relative velocity V_A when it reaches the surface of the planet, so that, by conservation of angular momentum,

$$V\, b_g = V_A\, r \, .$$

If there is no resisting medium which slows the particle before the collision (that is, the radius of the planet is measured to the top of the sensible atmosphere) then the loss of potential energy of the particle will equal its gain in kinetic energy:

$$V_A^2 = V^2 + \frac{2Gm_1}{r}$$

The second term on the right-hand side is the square of the planetary escape velocity, V_e, and these two equations can easily be re-arranged to give

$$b_g^2 = r^2\left[1 + \frac{V_e^2}{V^2}\right] = r^2\left[1 + \frac{2Gm_1}{rV^2}\right] \, . \tag{39}$$

Rotational symmetry gives the collision cross-section as

$$\sigma_c = \pi r^2\left[1 + \frac{V_e^2}{V^2}\right] = \sigma_G\left[1 + \frac{V_e^2}{V^2}\right] \tag{40}$$

where σ_G is the geometrical cross-section of the planet. Equation (40) is the same as equation (2) for the case of a particle of negligible mass and radius.

The Rutherford Scattering formulae, normally used for the Coulombic field, can equally well be used for the Newtonian field (Landau and Lifshitz, 1976). Substituting in the planetary gravitational potential the Rutherford Scattering impact parameter is:

$$b = \frac{Gm_1}{V^2} \cot(\chi_g/2) \quad . \tag{41}$$

Equations (39) and (41) then give

$$\cot(\chi_g/2) = \frac{rV^2}{Gm_1} \left[1 + \frac{2Gm_1}{rV^2}\right]^{\frac{1}{2}}$$

or, with $X = Gm_1/rV^2$,

$$\chi_g = 2 \arctan \left(\frac{X}{(1+2X)^{\frac{1}{2}}}\right) \tag{42}$$

which is the angular deflection producing a grazing impact. For a point-mass planet, χ_g would need to be 180°. However, for a real planet of finite size, χ_g is less than this: for example, the zenith attraction well-known to meteor astronomers can have a value of up to 17°. For Jupiter, χ_g may be of the order of 150°.

For a more exhaustive discussion of the application of the Rutherford Scattering impact parameter formula to the gravitational case, and the validity of the two, two-body approximation, see Ruppe (1966; p.146).

3.3 THE SPHERE OF INFLUENCE AND THE 'MINIMUM DEFLECTION'

Clearly there is no such thing as a 'minimum deflection' since the deflection tends to zero as the impact parameter becomes infinite. Here the term 'minimum deflection' (denoted χ_d) is applied to the smallest deflection produced by an encounter; an encounter is in turn defined as being a passage within the sphere of influence, of radius d. An angular deflection χ_d results from an encounter with impact parameter b=d, so that again using the Rutherford Scattering formula,

$$\chi_d = 2 \arctan \left[\frac{Gm_1}{dV^2}\right] = 2\arctan \left[\frac{Xr}{d}\right] \qquad (43)$$

and the radius of the sphere of influence, d, is required.

There have been a number of definitions used for the sphere of influence. The 'Hill Sphere' of the planet is the volume bounded by the Lagrangian points of the restricted three-body problem, of radius

$$d \approx R(m_1/M)^{1/3}$$

where R is the distance to the Sun, and

M is the mass of the Sun (Wetherill, 1980).

This can be derived by requiring that the perturbation of the particle's heliocentric orbit due to the planet be equal to the solar attraction; this is not the same as requiring that the planetary and solar attractions be equal (Ruppe, 1966).

Tisserand's more thorough investigation (Ruppe, 1966; p.153) renders

$$d = R[(1 + 3\cos^2 A)^{0.1}](m_1/M)^{0.4}$$

where A is the angle between the Sun-planet radius vector and the position angle of the particle relative to the planet. The bracketed term can vary from 1.00 to 1.15.

The definition used successively by Kislik (1964), Radzievskii (1967), Bandermann and Wolstencroft (1970), and Weidenschilling (1975 a) is:

$$d = 1.15\ R(^{m}1/M)^{1/3} \tag{44}$$

Since these definitions do not vary appreciably (Wetherill, 1980), the latter will be used here for ease of comparison with previous work. By substituting (44) into (43), the minimum deflection in an encounter (χ_d) can now be calculated.

3.4 THE MEAN AND ROOT-MEAN-SQUARE DEFLECTIONS IN AN ENCOUNTER

For an encounter where the planetary mass is large and the impact parameter is small, the deflection χ will be large; a small relative velocity would enhance the deflection (c.f. equation 41). In such an encounter all memory of the original trajectory will be lost, excepting that the Jacobi constant (Tisserand invariant) will be conserved (e.g. Kresak, 1979).

However, most passages are distant, so that the angular deflection is small and the new particle orbit is not substantially different from the original orbit. In a phase space described by some form of heliocentric angular coordinates, multiple encounters by the particle with different planets will result in a random walk of step length equal to the deflection. Multiple encounters thus result in a diffusion of the orbit until it no longer resembles the original particle orbit. The expectation value of the square of the ultimate deflection (χ_u) is just the sum of the squares of the individual deflections; i.e., for i encounters,

$$<\chi_u^2> = \sum_i \chi_i^2 \quad .$$

Öpik (1966 b) required that an accumulated $\chi_u = 90°$ should occur for a complete 'loss of memory' by the particle of its original orbit.

The above indicates that the mean and/or root-mean-square deflection per encounter would be of use in describing the diffusion of a particle orbit. In this section I derive these two parameters.

The Rutherford Scattering differential cross-section for deviation χ is:

$$d\sigma = \left[\pi\left(\frac{Gm_1}{V^2}\right)^2\right]\cos(\chi/2)\sin^{-3}(\chi/2)d\chi \tag{45}$$

(Landau and Lifshitz, 1976; p.53).

For a particular encounter the bracket is a constant, and the trigonometric part can be integrated (formula 322, p.568, CRC Handbook) to give a total cross-section as:

$$\sigma = \left[\pi\left(\frac{Gm_1}{V^2}\right)^2\right]\left[\sin^{-2}(\chi_{min}/2) - \sin^{-2}(\chi_{max}/2)\right] \tag{46}$$

Equation (46) can be used to determine an alternative expression to (40) for the collision cross-section, as follows. For an impact upon the planet, the minimum deflection required is $\chi_{min} = \chi_g$, given by (42). The maximum deflection corresponds to zero impact parameter, or $\chi_{max} = 180°$. Thus the collision cross-section is:

$$\sigma_c = \left[\pi\left(\frac{Gm_1}{V^2}\right)^2\right]\left[\sin^{-2}(\chi_g/2) - 1\right] . \qquad (47)$$

Substitution of the relevant planetary parameters and any required relative velocity shows that this renders the same result as found from (40). In all numerical work I have used (40) since it requires less computation.

A non-obvious error in Weidenschilling (1975a) should be pointed out here. Clearly his equations (12) and (13) do not lead to his equation (17), a factor of π being missing. Similarly, the π is missing from his equation (18). However, this is correct since his normalised units are equivalent to having a unit of time of $(1/2\pi)$ times the planet's orbital period, and a unit of area $(1/\pi)$. This follows from taking the unit of length as being the planet's semi-major axis, and the unit of velocity the planet's orbital velocity. Weidenschilling's equation (18) therefore gives an answer which is numerically correct, and for his units all is right if the factor of π is deleted from his equation (12).

Equations (45) and (46) can now be used to calculate the probability of obtaining any angular deflection χ within an interval $d\chi$. This depends only upon the relative velocity V for any particular planetary encounter since χ_d is a function of V. Since $\chi_{min} = \chi_d$ and $\chi_{max} = 180°$ this probability is

$$P(\chi|V)d\chi = \frac{d\sigma}{\sigma} = \frac{\cos(\chi/2)\sin^{-3}(\chi/2)}{\sin^{-2}(\chi_d/2)-1}\, d\chi . \qquad (48)$$

Note that this is the probability of such a deflection per encounter, and the definition of an encounter depends upon the definition of d (equation 44). By accepting a larger value of d, and hence a smaller minimum deflection χ_d, the probability of a certain large deflection per encounter is decreased markedly. The probability of such a deflection per unit time remains constant.

The mean deflection (χ_{BAR}) per encounter can now be found. This is

$$\chi_{BAR} = \frac{1}{\sigma_{nc}} \int_{\chi_d}^{\chi_g} \chi \; d\sigma$$

where σ_{nc} is the non-collisional cross-section:

$$\sigma_{nc} = \pi d^2 - \sigma_c$$

or

$$\sigma_{nc} = \left[\pi\left(\frac{Gm_1}{V^2}\right)^2\right]\left[\sin^{-2}(\chi_d/2) - \sin^{-2}(\chi_g/2)\right] \qquad (49)$$

since any $\chi \geqslant \chi_g$ results in a collision. This cross-section can be used in equation (7) to find P_D, the probability of a deflection.

After cancellation of the constants,

$$\chi_{BAR} = \left[\sin^{-2}(\chi_d/2) - \sin^{-2}(\chi_g/2)\right]^{-1} \int_{\chi_d}^{\chi_g} \chi \cos(\chi/2) \sin^{-3}(\chi/2)\, d\chi \qquad (50)$$

which can be integrated by parts (formula 5, p.542; formula 308, p.567; formula 322, p.568, CRC Handbook) to give:

$$\chi_{BAR} = \left[\sin^{-2}(\chi_d/2) - \sin^{-2}(\chi_g/2)\right]^{-1} \left[\begin{array}{l}\chi_d \sin^{-2}(\chi_d/2) - \chi_g \sin^{-2}(\chi_g/2) \\ +2\cot(\chi_d/2) - 2\cot(\chi_g/2)\end{array}\right] \qquad (51)$$

A typical value of $\chi_{BAR} \approx (1.8 \text{ to } 2.0)\chi_d$ is found.

Next the root-mean-square deflection χ_{RMS} is derived in a similar way. Since

$$\chi_{RMS}^2 = \frac{1}{\sigma_{nc}} \int_{\chi_d}^{\chi_g} \chi^2 \, d\sigma$$

an equation for χ_{RMS}^2 is derived which is identical to (50) except that the χ inside of the integral is replaced by χ^2. The equation can be integrated as before (using additionally formula 293, p.566, CRC Handbook) to get:

$$\chi_{RMS}^2 = \left[\sin^{-2}(\chi_d/2) - \sin^{-2}(\chi_g/2)\right]^{-1}$$

$$\left[\begin{array}{l} \chi_d^2 \sin^{-2}(\chi_d/2) \quad - \chi_g^2 \sin^{-2}(\chi_g/2) \\ +4\chi_d \cot(\chi_d/2) \quad - 4\,\chi_g \cot(\chi_g/2) \\ +8\,\ell n(\sin(\chi_g/2)) - 8\,\ell n(\sin(\chi_d/2)) \end{array}\right] \quad (52)$$

In Weidenschilling (1975 a) the equivalent to this (his equation 27) contained an error.

A typical value of $\chi_{RMS} \approx (2.6 \text{ to } 2.8)\chi_d$ is derived.

3.5 PROBABILITY OF EJECTION

Using the deflection range χ_d to χ_g it is now possible to calculate the probability of an encounter resulting in the particle being ejected from the solar system.

The planet is considered to be a scattering centre which is momentarily on a circular orbit of radius $\bar{R}$, so that the circular velocity is:

$$V_o = \left[\frac{GM}{\bar{R}}\right]^{\frac{1}{2}} \quad (53)$$

At this solar distance the critical heliocentric velocity for ejection from the solar system is

$$V_c = \left[\frac{2GM}{\bar{R}}\right]^{\frac{1}{2}} \tag{54}$$

so that $V_c^2 = 2V_o^2$.

As discussed previously, in the encounter it is assumed that V_{RC}, the relative velocity of the particle and the planet taken to be on a circular orbit, is conserved in magnitude but not in direction. In Figure 4a, V_{B2} is the particle velocity and θ is the encounter angle, given by equation (37) with $\gamma_1 = 0$ since a circular orbit is assumed. Thus

$$\cos\theta = \cos\gamma_2 \cos(\alpha_1 \pm \alpha_2) \tag{55}$$

and V_{RC} is given by

$$V_{RC}^2 = V_2^2 + V_o^2 - 2V_2V_o \cos\theta \tag{56}$$

Figure 4b shows the geometry after the encounter. A deflection χ results in V_{RC} being conserved with its orientation changed from ϵ to ϵ', and thus its heliocentric velocity is changed to V'_{B2}. This new velocity will not in general be in the same plane as V_{B2}, and it is assumed that all orientations of the new relative velocity vector V_{RC} are equally likely for a given ϵ' (i.e. the scattering is rotationally symmetric).

This is more clearly shown in Figure 5. A deflection χ results in a new direction of V_{RC} which may be anywhere on the circle ABCD. An ejection requires

$$V'_{B2} \geqslant V_c$$

FIGURE 4a

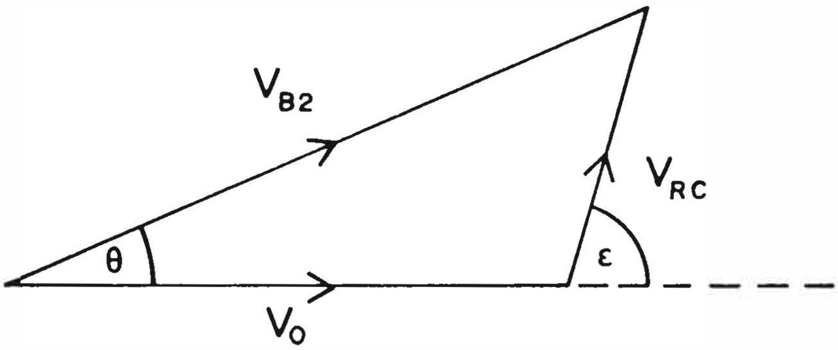


FIGURE 4b

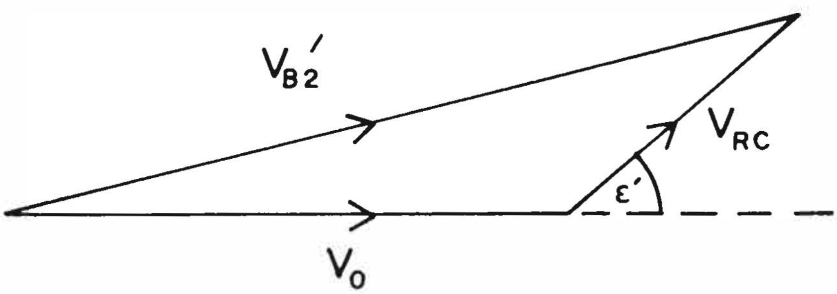


FIGURE 4

Pre- and post-encounter geometry

FIGURE 5

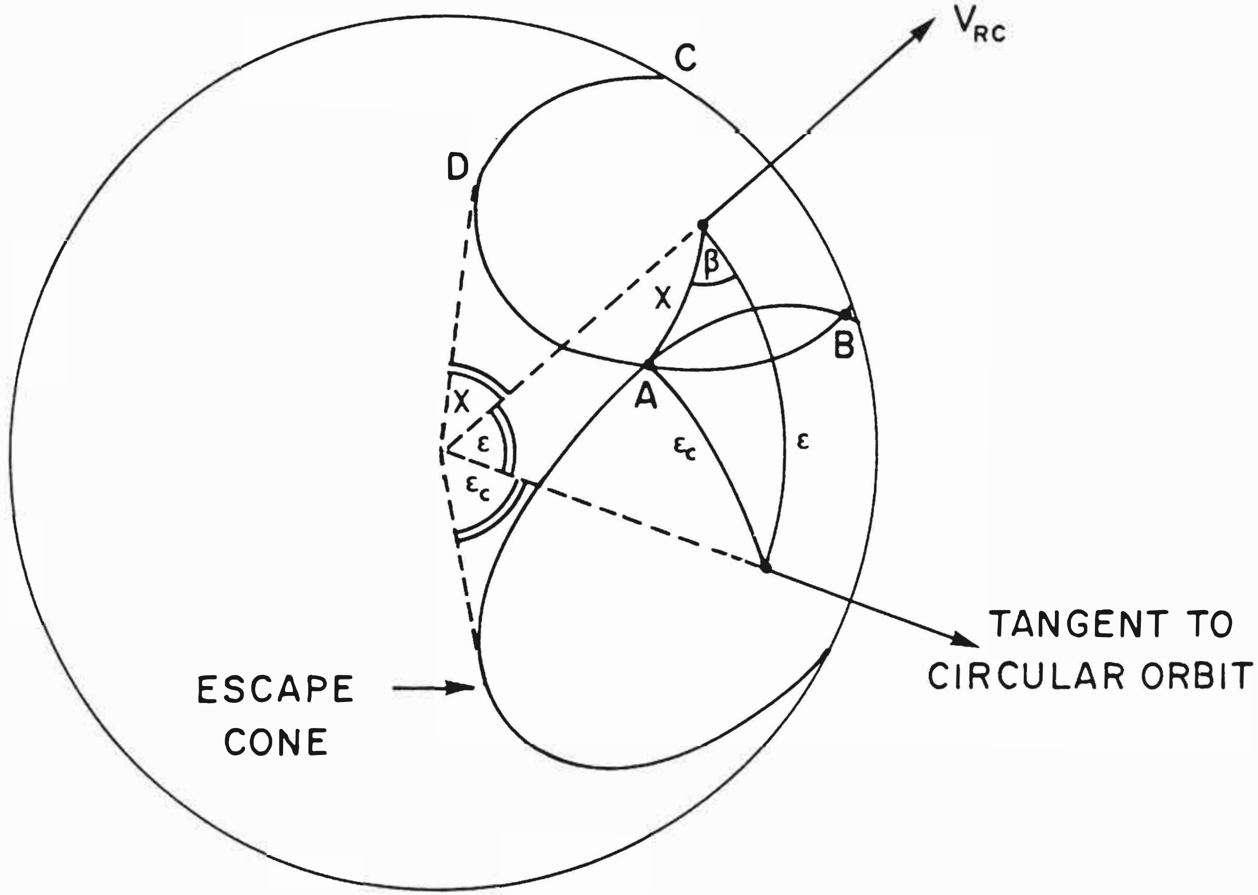


FIGURE 5

Deflection in an encounter, and the escape cone and this results in an 'escape cone', symmetric about the tangent, which is described by a half-angle ϵ_c. This angle is determined by setting $V'_{B2} = V_c$ and $\epsilon' = \epsilon_c$ in Figure 4b, so that

$$\cos \epsilon_c = (V_c^2 - V_{RC}^2 - V_o^2)/2V_{RC}V_o$$

or

$$\cos \epsilon_c = (V_o^2 - V_{RC}^2)/2V_{RC}V_o \tag{57}$$

and any $\epsilon' \leqslant \epsilon_c$ results in an ejection.

Similarly from Figure 4a,

$$\cos \epsilon = (V_{B2}^2 - V_{RC}^2 - V_o^2)/2V_{RC}V_o \tag{58}$$

and a deflection of at least $(\epsilon - \epsilon_c)$ is necessary for the particle to attain an hyperbolic heliocentric velocity. The probability of this is now required.

Any new vector V_{RC} which lies inside of the escape cone will result in an ejection. In Figure 5 a particular deflection χ will give an ejection if the new direction of V_{RC} lies anywhere along the arc AB. Since it has been assumed that all orientations for a given χ are equally likely, this means that the probability of an ejection in this case is just the length of arc AB divided by the perimeter of circle ABCD. If β is the half-angle of arc AB, the probability of ejection here (i.e. the probability of an infinite heliocentric distance resulting) is:

$$P(\infty) = \beta/\pi \quad . \tag{59}$$

The angle β can be calculated since the angular lengths $(\epsilon, \epsilon_c, \chi)$ of its spherical triangle are known, so that:

$$\cos\beta \sin\chi \sin\epsilon = \cos\epsilon_c - \cos\chi \cos\epsilon$$

or

$$\beta = \arccos\left[\frac{\cos\epsilon_c - \cos\chi\cos\epsilon}{\sin\chi \sin\epsilon}\right] \quad . \qquad (60)$$

To get the total ejection probability for a given encounter it is necessary to integrate $P(\infty)$ over all possible deflections (all possible impact parameters). To do this the minimum and maximum possible deflections, χ_{min} and χ_{max}, are required.

It has been seen that a deflection of at least $(\epsilon-\epsilon_c)$ is needed, which would normally be the value of χ_{min}. However, it is possible that the deflection at the edge of the sphere of influence (χ_d) might be greater than $(\epsilon-\epsilon_c)$. Therefore,

$$\begin{aligned} \chi_{min} &= \epsilon-\epsilon_c \qquad (\chi_d < \epsilon-\epsilon_c) \\ \chi_{min} &= \chi_d \qquad (\chi_d > \epsilon-\epsilon_c) \end{aligned} \qquad (61)$$

Note that it is still possible for particles whose original orbits are near-parabolic to be ejected from the solar system although their closest approach is $(b > d)$: only a very small deflection is necessary for those particles to attain a hyperbolic heliocentric orbit. Thus for particles of large a the ejection probability derived here would be a lower limit.

From Figure 5 it is also apparent that it is possible to 'over-deflect' a particle: if $\chi > (\epsilon+\epsilon_c)$ then the particle is deflected past the escape cone and ejection is not possible.

This is in fact one way in which near-parabolic comets may be captured into short-period orbits by a close encounter with one of the giant planets (Everhart, 1968, 1972, 1973). Similarly if $\chi > \chi_g$ then an impact occurs and ejection is impossible; the maximum possible deflection for an ejection is therefore:

$$\begin{aligned} \chi_{max} &= \epsilon+\epsilon_c \qquad (\chi_g > \epsilon+\epsilon_c) \\ \chi_{max} &= \chi_g \qquad (\chi_g < \epsilon+\epsilon_c) \end{aligned} \tag{62}$$

An additional constraint is that for some trajectories the minimum deflection necessary for an ejection is in fact greater than the deflection producing a grazing impact. Equally well the maximum deflection producing an ejection could be less than the deflection produced at the edge of the sphere of influence. Thus for a non-zero ejection probability it is required that:

$$\begin{aligned} &\chi_{min} < \chi_g \\ \text{and} \quad &\chi_{max} > \chi_d \quad . \end{aligned} \tag{63}$$

Subject to (61,62,63) the probability of ejection for a given encounter (given V_{RC}, ϵ) is obtained by integrating equation (60) over all possible differential cross-sections. This probability is:

$$P(\infty|V_{RC},\epsilon) = \int_{\chi_{min}}^{\chi_{max}} (\beta/\pi) \frac{d\sigma}{\sigma}$$

The differential cross-section is given by (48) so that:

$$P(\infty|V_{RC},\epsilon) = [\pi(\sin^{-2}(\chi_d/2) - 1)]^{-1} \cdot \int_{\chi_{min}}^{\chi_{max}} \beta \cos(\chi/2)\sin^{-3}(\chi/2)\,d\chi \tag{64}$$

with β from equation (60).

This probability can now be used to find an 'effective ejection cross-section', σ_e. The encounter cross-section is:

$$\sigma_d = \pi d^2 \tag{65}$$

and σ_e is defined here as:

$$\sigma_e = \sigma_d \, P(\infty|V_{RC},\epsilon) \quad . \tag{66}$$

Equations (65) and (66) give cross-sections for encounter and ejection respectively, which can be used in equation (7) to determine the probability per unit time of these events occurring.

The integral in equation (64) must be evaluated numerically. This can be performed easily using Simpson's Rule, as follows.

3.5.1 <u>The Ejection Integral Using Simpson's Rule</u>

The integral in equation (64), with β given by (60), needs to be found numerically. The standard method for performing such a quadrature is the use of Simpson's Rule, which is applicable where the polynomial is of the third-degree or less. (See, for example, Kopal, 1955; Conte, 1965; Dorn and McCracken, 1972).

Here Simpson's One-Third Rule has been used, viz:

$$I = \int_{x_1}^{x_1+2h} f(x)\,dx = \frac{h}{3}[f(x_1)+4f(x_1+h)+f(x_1+2h)] - Erf \quad .$$

The error function (Erf) is of the order of h^5 and so is small as long as the region of interest is split up into a sufficiently large number of slices (n), each of width 2h. The integral is then given by the sum of the n values of I.

The number of slices n can be quite small without any great loss of precision: in most cases the total error is dominated by the computational round-off error rather than the imprecision error (Erf) for $n \geqslant 100$. Due to this consideration, and also because in the present situation (i) precision is not of paramount importance, and (ii) this integration is nested within another summation (equation 7), it was decided to use $n = 45$. Thus the maximum possible value of $d\chi$ is $\Delta\chi = h = 2°$, but will generally be substantially smaller.

By trial it was found that, for the function in question, changes well below 1% in the value of $P(\infty|V_{RC},\epsilon)$ resulted from increasing n above 45. The total computation time for an arbitrary particle orbit is also kept within a reasonable limit using this number of iterations.

3.6 ORBITAL ENERGY CHANGE PER ENCOUNTER

The orbital energy of a particle of mass m and semi-major axis a is:

$$E = -\frac{GmM}{2a}$$

Rather than carry the constant I shall define as a measure of the orbital energy the quantity

$$K = - 1/a \tag{67}$$

so that

$$E = \frac{GmMK}{2} \quad . \tag{68}$$

An ejection upon a hyperbolic heliocentric orbit corresponds to K becoming positive. A parabolic orbit has $K = 0$, and a bound elliptic orbit has a lower energy than this (i.e. K negative). An increase in semi-major axis therefore corresponds to an increase in orbital energy.

One way to calculate the mean energy change in an encounter would be to derive an expression for the probability of a certain new energy K, $P(K|V_{RC},\epsilon)$. This could easily be performed, in the same way as the derivation of $P(\infty|V_{RC},\epsilon)$. The only difference would be that the critical (escape) angle ϵ_c would be replaced by an angle ϵ_k representing the minimum deflection necessary to reach the required energy K. This angle would be defined by an equation similar to (57), replacing V_c by V_k, this being the velocity at radial distance R of an orbit having energy K:

$$V_k = \left[GM\left(\frac{2}{R} + K\right)\right]^{\frac{1}{2}} \quad . \tag{69}$$

However, an added complexity is that to derive the mean energy change per encounter it would be necessary to integrate $P(K|V_{RC},\epsilon)$ over narrow bands $d\epsilon$ centered on ϵ_k for each possible value of K; that is, the probability of finding an energy K within an interval dK would be required. Thus a double integral would result (c.f. equation 36 in Weidenschilling, 1975 a), whose numerical evaluation would be prohibited by the necessary computation time.

A much more simplistic approach is followed here in order to get a rough estimate of the rate of orbital energy change due to close encounters for comparison with other dynamical effects.

For all possible encounter geometries the maximum and the minimum relative velocities are selected, and the average of these two is taken to be a measure of the mean encounter velocity, V_m. The mean planetary orbital velocity $\bar{V}_{B1}$ is estimated from the velocity which would occur if it followed a circular orbit of radius equal to its semi-major axis:

$$\bar{V}_{B1} = [GM/a_1]^{\frac{1}{2}} \tag{70}$$

The particle velocity at this distance can also be calculated:

$$V_{B2} = [GM(\frac{2}{a_1} - \frac{1}{a_2})]^{\frac{1}{2}} \tag{71}$$

The geometry is now similar to that in Figures (4a) and (4b). Before the fictitious encounter the encounter angle is given by:

$$\cos \epsilon = \left[\frac{V_{B2}^2 - \bar{V}_{B1}^2 - V_m^2}{2\,\bar{V}_{B1} V_m} \right] \tag{72}$$

A measure of the deviation is taken to be χ_{RMS}, so that the trajectory after such an encounter would be described by:

$$\epsilon' = \epsilon \pm \chi_{RMS} \tag{73}$$

Note that this assumes that the new orbital velocity vector V'_{B2} is in the same plane as the velocity V_{B2} before the encounter (i.e. in Figure 5, $\beta = 0°$ or $180°$) so that this represents an overestimate of the orbital energy change. The new velocity V'_{B2} is found by again assuming that the velocity relative to the planet, V_m here, is changed in direction but not magnitude. Thus

$$V'^2_{B2} = \bar{V}^2_{B1} + V^2_m + 2\bar{V}_{B1}V_m\cos\epsilon' = GM\left[\frac{2}{a_1} - \frac{1}{a_2'}\right] \qquad (74)$$

renders the semi-major axis of the particle's new heliocentric orbit, and the change in orbital energy is:

$$\Delta K = \frac{1}{a_2} - \frac{1}{a_2'} \qquad (75)$$

A positive value of ΔK represents an increase in energy. Two values for ΔK from equation (75) result; combining equations (71 to 75) these values are:

$$\Delta K = \frac{2\bar{V}_{B1}\ V_m}{GM}\left[\cos(\epsilon \pm \chi_{RMS}) - \cos\epsilon\right] \ . \qquad (76)$$

A measure of the overall energy change per encounter is:

$$(\Delta K_+ + \Delta K_-) = \frac{2\bar{V}_{B1}\ V_m}{GM}\left[\cos(\epsilon - \chi_{RMS}) + \cos(\epsilon + \chi_{RMS}) - 2\cos\epsilon\right] \qquad (77)$$

The fractional change in orbital energy per encounter is given by (77) divided by (67), and the fractional change per year is found by multiplying this result by P_D (the probability of an encounter, found from equation 7 using equation 49 for the cross-section). Thus

$$\frac{-\dot{E}}{E} = + \frac{2\, a_2\, P_D\, \bar{\bar{V}}_{B1}\, V_m}{GM} \left[\cos(\epsilon - \chi_{RMS}) + \cos(\epsilon + \chi_{RMS}) - 2\cos\epsilon \right] \qquad (78)$$

is used here as a crude measure of the rate of orbital energy change due to planetary encounters.

The sign of (78) has been chosen such that, again, a positive value represents an increase in orbital energy.

The value derived from the orbital energy change is taken to be an order of magnitude estimate only, for comparison with other dynamical effects. Particular care needs to be taken when the particle's perihelion or aphelion distance is close to the planet's semi-major axis. This is noted upon in more detail in chapter 5.

CHAPTER 4

FACTORS LIMITING THE LIFETIMES OF METEOROIDS

4.1 INTRODUCTION

Using the techniques described in Chapter 2 to determine the frequency of planetary encounters for a particle in an arbitrary heliocentric orbit, and Chapter 3 to deduce the results of such an encounter, it is now possible to define lifetimes for a particular body against certain events occurring. The meaning of such a lifetime (τ) is that after a time τ the fraction (f) of particles left in the original orbit will be, for a Poisson distribution of encounters,

$$f = \exp(-t/\tau) \qquad (79)$$

The relevant lifetimes (or time-scales) are those for planetary collision (τ_C), gravitational deflection (τ_D), or ejection from the solar system (τ_E). If the probabilities (per unit time) of such events are P_C, P_D and P_E respectively, then

$$\tau_C = 1/P_C$$

$$\tau_D = 1/P_D \qquad (80)$$

$$\tau_E = 1/P_E$$

A deflection results only in a new elliptical orbit, so that the interplanetary meteoroid population is not depleted, only re-arranged. However, a planetary collision or ejection from the solar system removes a meteoroid from the complex; a loss time-scale (τ_L) is hence of use, defined as

$$\tau_L = 1/(P_C + P_E) \qquad (81)$$

Clearly τ_D is useful in predicting the production rate of sporadic meteors from streams; however τ_D is <u>not</u> the time-scale for sporadic production since it is possible that the deflection produced in a planetary encounter will be insufficient to produce a significant change in the orbit of a particular meteoroid compared to the stream. In the next section of this chapter I show how τ_D can be used to determine a time-scale for sporadic meteoroid production, τ_S.

Although τ_L describes the rate at which meteoroids are lost in planetary close encounters, there are many other factors which limit the lifetimes of interplanetary particles. These factors include radiative effects (Poynting-Robertson, Yarkovsky-Radzievskii, differential Doppler, radiation pressure), electromagnetic effects (Lorentz scattering, Coulomb drag), rotational bursting, and collisions with other particles. In the subsequent sections of this chapter each is reviewed and its importance to the dynamical evolution of meteoric particles (size range ~ 100 μm - 1cm) is assessed.

4.2 SPORADIC METEOR PRODUCTION

In order to define a time-scale for the production of sporadic meteors by the random planetary disruptions of meteor streams, it is clearly necessary to differentiate between these two types of meteoric particle.

When making surveys of the characteristics of meteoroid orbits, by whatever observational technique, it is usual to use as a criterion for deciding upon stream membership the D-parameter of Southworth and Hawkins (1963). For two orbits A and B, which may represent two independent meteors or a measured meteor and the mean orbit of a stream, the D-parameter is defined as, for low inclinations:

$$
\begin{aligned}
[D(A,B)]^2 &= [e_B - e_A]^2 + [q_B - q_A]^2 \\
&+ \left[2 \sin\left(\frac{i_B - i_A}{2}\right)\right]^2 \\
&+ \sin i_A \sin i_B \left[2 \sin\left(\frac{\Omega_B - \Omega_A}{2}\right)\right]^2 \\
&+ \left[\left(\frac{e_A + e_B}{2}\right) 2 \sin\left(\frac{\Omega_B + \omega_B - \Omega_A - \omega_A}{2}\right)\right]^2
\end{aligned}
\qquad (82)
$$

where e is the orbital eccentricity,

q is the perihelion distance,

i is the inclination,

Ω is the longitude of the ascending node,

and ω is the argument of perihelion.

Southworth and Hawkins (1963) have discussed the scaling factors used in this definition, as have several other authors. There have been other definitions of criteria for orbit differentiation (e.g. Drummond, 1979; 1981), but for the present rudimentary use, (82) is entirely adequate.

When using this criterion to discriminate between shower and sporadic meteors, some maximum value of D must be chosen. Southworth and Hawkins (1963) decided upon $D \leqslant 0.20$, although other authors have used a lower limit. Since a set of measured orbital elements contains measurement errors in addition to real orbital variations, the value of D used in an orbit survey must reflect the expected uncertainties in the measurements (see Gartrell and Elford, 1975, for a discussion). Although an argument could be made for a lower value, I will use $D = 0.20$ as the limit here.

There are two ways in which I have used the D-criterion to get a crude estimate of the characteristic time-span for sporadic meteor production from streams. The first is as follows. In Figure 5, since the perturbing planet is close to the ecliptic plane a deflection χ produces a mean change in inclination of the same order, especially so if $|\beta| \simeq 90°$. If no other change in the orbital elements were produced then equation (82) is very much simplified and the difference between perturbed orbit B and unperturbed orbit A is

$$D = 2 \sin\left(\frac{i_B - i_A}{2}\right)$$

or, with $\chi \approx (i_B - i_A)$ and $D = 0.2$,

$$\chi = 2 \arcsin(0.1)$$

so that a deflection of $\sim 10°$ is required. This could be regarded as the necessary accumulated deflection for the production of sporadic meteors from a stream source, and can be compared with the $\chi_u = 90°$ required by Öpik (1966b) for complete 'loss of memory' by the particle of its original path (chapter 3). Since the expectation value of the net deflection accumulates as the sum of the squares, one deflection of 10° or a hundred deflections of 1° would be necessary. The time-scale for an accumulated deflection of 10° is therefore:

$$\tau_A = 100/\sum P_D \chi^2_{RMS} \tag{83}$$

where the summation is over all of the planets intersected by the particle.

I will now move on to an alternative method for estimating this time-scale. Southworth and Hawkins (1963) pointed out that a measure of the difference between two orbits is the perturbation required to transform one orbit into the other. Since a perturbation is just a force applied for a certain length of time, it can be measured by the change in velocity that it produces. As discussed by Southworth and Hawkins (1963), since the D-criterion encompasses all of the orbital elements it is a measure of the total perturbation which has changed an orbit, or the total velocity change produced. They find that for an orbit with an eccentricity not close to 1, D is about 1.5 times the mean velocity increment (Δv) in units of the circular velocity (v). Thus

$$D \simeq \frac{3}{2}\frac{\Delta v}{v} \qquad (e \lesssim 0.85)$$

or, for D = 0.20, a sporadic meteor is produced from a stream if

$$\frac{\Delta v}{v} \simeq 0.13$$

Since

$$E = -\frac{1}{2}v^2$$

is the orbital energy per unit mass,

$$\frac{\Delta E}{E} \simeq 2\frac{\Delta v}{v}$$

and so I will adopt a criterion for sporadic meteor production of

$$\frac{\Delta E}{E} \simeq 0.25$$

This can now be used in conjunction with the rate of energy change calculated from equation (78) to find the time-scale for sporadic production:

$$\tau_S \simeq \frac{E}{4\dot{E}} \tag{84}$$

The result from equation (84) will be an overestimate of the time-scale since in deriving (78) only the difference in orbital energies between the forward- and backward-scattered particles was used. Actually all scattered particles suffer changes to their energy and orbital elements; however (84) will suffice in the present situation.

The values of τ_A and τ_S calculated from equations (83) and (84) represent two alternate time-scales for the production of sporadic meteors by planetary disruptions of meteor streams. As noted above, τ_S is an overestimate, but the derivation of τ_A is also somewhat crude. Their values should, however, give some idea of the rate at which the planets disrupt streams, for comparison with the other processes at work.

4.3 RADIATIVE FORCES ON METEOROIDS

The various influences upon meteoroidal orbits due to the solar radiation field have been the subject of intensive study and I will only briefly mention them here, for the purposes of finding the relevant time-scales. An excellent and exhaustive review is that of Burns et al (1979).

4.3.1 Radiation Pressure

Clearly the absorption of solar radiation results in an outward force due to the momentum carried by the photons. The impulse will depend upon the scattering coefficients of the particle, the solar distance, and also upon the particle's radial velocity (because of the Doppler shift in the absorbed wavelengths). Radiation pressure is of extreme importance for particles smaller than ~ 1 μm (Leinert, 1975) although Burns et al (1979) find that, contrary to previous conclusions, submicron dust is not easily blown out of the solar system.

For larger particles it is convenient to include the effect of radiation pressure with the Poynting-Robertson force.

4.3.2 Poynting-Robertson Effect

The Poynting-Robertson effect (Poynting, 1904; Robertson, 1937; Wyatt and Whipple, 1950) results in an interplanetary particle slowly spiralling into the sun. Although formulated by Robertson (1937) using the metric of special relativity, Burns et al (1979) have shown that a classical derivation is possible.

As mentioned above, the solar radiation field results in an outward force upon an orbiting particle due to the photons which it scatters or absorbs. Those absorbed photons have an additional role to play in that they heat the particle, their energy being re-radiated at longer wavelengths. However, relative to the particle's orbital motion the forward-emitted photons are blue-shifted by the Doppler effect, the converse being true for the photons emitted in the opposite direction. This results in more momentum being lost from the forward-pointing hemisphere, and a braking effect upon the particle. The gradual loss of orbital energy in this way causes it to spiral into the sun. The decelerating drag is due to the difference between the incoming and outgoing momenta. The outward radiation pressure force can easily be combined with the inward Poynting-Robertson force, although the former is small for the particles of interest here (100 μm - 1 cm) since these have a large mass-to-surface-area ratio (Dohnanyi, 1978; Figure 17).

There are a number of expressions for the Poynting-Robertson lifetime, depending upon whether the initial orbit is circular (the effect rapidly decreases the eccentricity), the actual infall distance required, and other factors. I shall use a definition for the Poynting-Robertson time-scale (τ_{PR}) based upon equation (54) of (Dohnanyi, 1978). This is

$$\frac{da}{dt} = -\left[\frac{3E_{\odot}\nu}{16\pi c^2 s\rho}\right] \frac{(2+3e^2)}{a(1-e^2)^{3/2}}$$

where $E_{\odot}$ is the total solar output in Watts;

ν is the fraction of the momentum of the incident photons transfered to the meteoroid ($\nu = 1$ for pure absorption, $\nu = 2$ for direct reflection);

c is the velocity of light;

s is the meteoroid radius;

and ρ is the meteoroid density.

Putting in the relevant values and with $\nu = 1$, $\rho = 10^3$ kg m^{-3}, this becomes

$$\dot{a} = \frac{2.5 \times 10^8}{s} \times \frac{(2+3e^2)}{a(1-e^2)^{3/2}} \tag{85}$$

where s is expressed in metres.

From the expression for orbital energy (equation 68),

$$\frac{\dot{E}}{E} = -\frac{\dot{a}}{a}$$

and I shall define the Poynting-Robertson time-scale as

$$\tau_{PR} = \frac{E}{\dot{E}} = -\frac{a}{\dot{a}} \tag{86}$$

or, using (85) with a expressed in A.U. and the year as the unit of time,

$$\tau_{PR} = \frac{2.8 \times 10^9 \; s \; a^2 \; (1-e^2)^{3/2}}{(2 + 3e^2)} \tag{87}$$

where again s is in metres. The reason for this choice of definition for τ_{PR} is that it involves the fractional rate of change of orbital energy (c.f. equations 78 and 84).

Equation (87) indicates that τ_{PR} increases linearly with particle size. About the shortest-lived meteoroid in the domain of interest would be a Geminid member ($a \approx 1.4$ A.U., $e \approx 0.9$) of diameter 100 μm: it would have a lifetime $\tau_{PR} \approx 10^4$ years or somewhat less, due to its small perihelion distance. Generally the expected lifetimes from equation (87) vary from 10^5 to 10^7 years. Note that although a factor of a^2 appears in (87), the initial semi-major axis is comparatively unimportant: the dominant factor is the initial perihelion distance.

An additional energy source which enhances the Poynting-Robertson effect is the absorption of solar wind particles. This has sometimes been termed the 'pseudo Poynting-Robertson effect' since it acts in the same way: the outward pressure is insignificant compared to the radiation pressure and the drag is due to the difference in momentum of the Doppler-shifted re-emitted photons (Dohnanyi, 1978). This may cause an enhancement of the order of 20% in the Poynting-Robertson deceleration (Whipple, 1967; Carpenter and Pastusek, 1967). The effect of the solar wind in this connection is especially important for submicron dust particles (Burns et al, 1979), but is of limited significance to meteoroids larger than 100 μm. This has been investigated in detail by Mukai and Yamamoto (1982).

Finally it is useful to note that although Burns et al (1979) expanded the previous work on the Poynting-Robertson effect to include real scattering laws, their treatment was still limited to spherical particles. A start upon the problem of irregularly-shaped particles has been made by Kerker (1980).

4.3.3 Yarkovsky-Radzievskii Effect

An additional consequence of the momentum lost by a meteoroid through re-emitted photons is introduced if the meteoroid is spinning. The temperature distribution across its surface will not be uniform since the 'evening hemisphere', having just been exposed to the sun, will be warmer than the 'morning hemisphere'. Therefore there will be an asymmetry in the radiated momentum, and a particle which spins in the same sense as its orbital motion will be accelerated. The sign of this force (acceleration or deceleration) will depend upon the spin direction. The Yarkovsky-Radzievskii effect hence counteracts the Poynting-Robertson drag for prograde-spinning particles, but enhances it for a retrograde spin. The effect has been re-discovered on a number of occasions (Öpik, 1951; Radzievskii, 1952; Petersen, 1976; Burns et al, 1979).

Clearly this will be a three-dimensional diffusive influence, the magnitude and direction of the force varying as the spin rate and orientation of the particle is changed by inter-particle collisions and other factors. The time-scale of the effect (Radzievskii, 1952; Dohnanyi, 1978) is:

$$\tau_{YR} \simeq \frac{30\ s\rho^{3/2}\ a^3}{P^{1/2}} \tag{88}$$

where s is the particle radius in metres;

ρ is the particle density in kg m^{-3};

a is the radius of its orbit (in A.U.), taken to be circular;

and P is the particle spin period in seconds.

It is found that for particles larger than dust grains (i.e. meteoroids 100 μm - 1 cm of interest here) the Yarkovsky-Radzievskii force is significant and may dominate the Poynting-Robertson force (Burns et al, 1979).

From the evidence of the initial widths of radar meteor trails, Hawkes and Jones (1978) have made an argument for spin rates of the order of 10^3 revolutions per second for meteoroidal particles. With this spin rate and again taking a 100 μm Geminid meteor as an example, equation (88) renders $\tau_{YR} \simeq 6 \times 10^3$ years. This is not significantly different from the Poynting-Robertson lifetime τ_{PR}. It must be remembered that the Yarkovsky-Radzievskii effect will lead to a three-dimensional random walk rather than a simple infall to the sun (Dohnanyi, 1978) and hence could be important in the production of sporadic meteors from streams. However there is some doubt as to whether it is of any significance for objects of meteoroid dimensions, or indeed any smaller than ~ 1 metre, since solar radiation pressure will cause the spin axis to precess and the force then time-averages to zero (Slabinski, 1979; Burns et al, 1979). Therefore I shall take the Yarkovsky-Radzievskii force to be of limited significance compared to the Poynting-Robertson drag in discussions of the lifetime of sporadic meteoroids.

4.3.4 Differential Doppler Effect

A subtle influence upon the motion of an interplanetary particle, which was not identified until 1975, is the differential Doppler effect. This has been discussed by Burns et al (1979) and is but briefly mentioned here since it is only of importance for a meteoroid close to the solar surface.

When a particle is close to the sun, the latter can no longer be approximated as a point-source of radiation (Guess, 1962). Due to the solar rotation, one half of this extended source will be approaching the particle, with a resultant blue-shift in the received photons, and the radiation from the other half will be red-shifted. The drag force resulting is always smaller than the Poynting-Robertson drag, and is insignificant at all solar distances of interest here.

4.4 LORENTZ SCATTERING

Interplanetary grains are thought to be at a potential of the order of 1 to 10 volts due to impinging charged solar wind particles and also photoemission caused by the solar ultra-violet flux (Mukai, 1981; Lafon et al, 1981). This charging leads to a perturbative force upon the particles since not only are they moving relative to the magnetic field carried by the solar wind, but additionally the magnetic field is changing at a fixed point in space as the wind moves away from the sun (Parker, 1964; Morfill and Grün, 1979; Consolmagno, 1979, 1980). The particles thus experience a Lorentz force which rapidly varies as the magnetic field changes in intensity and direction.

The Lorentz force can be of the same magnitude as the gravitational attraction of the sun for high-speed submicron particles (Levy and Jokipii, 1976) but its major significance for interplanetary particles is in the size range from 1 to 10 μm (Morfill and Grün, 1979; Mukai, 1981; Mukai and Giese, 1984). The Lorentz scattering force can be up to 10% of the Poynting-Robertson force for a 30 μm dust grain, but is unimportant for particles of radius > 100 μm (Consolmagno, 1979, 1980). Its influence upon the Keplerian orbits of dust particles will be diffusive in nature (Barge et al, 1982a,b) and it is thought to be responsible for the overall distribution of the zodiacal cloud of dust (Mukai and Giese, 1984).

From the intensive study of Lorentz scattering of interplanetary dust which has been carried out over the past five years it has become clear that this force is of limited significance to meteoroidal particles, and hence is ignored here.

Coulomb drag is also negligible (Leinert et al, 1983).

4.5 ROTATIONAL EFFECTS

The particle spin described in section 4.3.3 occurs due to the influence of interparticle collisions and also the solar radiation field. The former is a fairly obvious cause of spin and has received much attention as the source of asteroidal rotations (Gehrels, 1979). The latter arises because asymmetries either in the geometrical shape or the refractive index of an interplanetary particle will result in a torque being imposed by the scattered photons (see Dohnanyi, 1978). If this torque continues to act in the same sense then the spin rate will increase until the angular velocity is such that the particle breaks asunder. This has been termed 'rotational bursting' (Paddack and Rhee, 1976). For materials whose angular acceleration is not damped in any way it has been estimated that a life-span of the order of 10^5 years is applicable to a 1 cm meteoroid. For metallic particles, or chondritic meteoroids containing metallic inclusions, the spin is magnetically damped and a lifetime of $\sim 10^6$ years at 1 A.U. results (Paddack and Rhee, 1976). These periods would be shorter than the relevant Poynting-Robertson lifetimes but have been disputed in view of other damping mechanisms (Sparrow, 1975). As pointed out by Dohnanyi (1978), it is difficult to quantitatively estimate the time-scale associated with rotational bursting; however this is of little importance since in the next section it is shown that in the size range of interest (100 μm - 1 cm) the dominant effect, limiting the lifetimes of these meteoroidal particles, is in fact catastrophic interparticle collisions.

It is not really possible that this windmill-like spinning up is of relevance to the lifetimes of meteoroidal particles on account of the particle's rotational kinetic energy prior to a collisional. For reasonable spin rates (e.g. Hawkes and Jones, 1978) and encounter velocities, the translational kinetic energy is greater than its rotational counterpart by a factor of $\sim 10^6$. The contrary suggestion of Dohnanyi (1978) hence appears to be incorrect.

An alternative spin-up mechanism, with the spin axis being parallel to the motion of the particle rather than the sun-particle direction, was suggested by Radzievskii (1954).

4.6 INTER-PARTICLE COLLISIONS

4.6.1 Introduction

An understanding of the role of inter-particle collisions in the evolution of meteoroids and the zodiacal cloud has only been possible since spacecraft observations of the spatial density of particles have been available. The last fifteen years have seen an intensive effort in this area, and present models depend upon several poorly-known parameters such as the density of interplanetary particles and their physical strength.

It was not until 1975 that Zook and Berg showed that the lifetimes of particles larger than 100 μm are limited by collisions within 1 A.U., so that the Poynting-Robertson drag and other loss mechanisms described earlier in this chapter are not dominant for the majority of meteoroids. This conclusion has since been confirmed by other researchers (e.g. Le Sergeant D'Hendecourt and Lamy, 1981; Trulsen and Wikan, 1980).

The results of a collision can be considered in a number of distinct regimes (e.g. the discussions by Hughes, 1978, and Dohnanyi, 1978). If the relative kinetic energy of two intersecting particles is small (e.g. a low-mass dust grain collides with a meteoroid at a low relative velocity) then the result will be an erosion: some of the meteoroid will be chipped away. Above a certain limit the collision energy is sufficient to fragment the meteoroid into numerous small pieces: a catastrophic collision. At the other end of the scale, impinging solar wind particles will cause sputtering, gradually eroding the meteoroid (Mukai and Schwehm, 1981).

It is known that the erosive processes cause physical decay of meteoroids much more slowly than the destructive impacts, and so these are usually ignored (Dohnanyi, 1972; Le Sergeant D'Hendecourt and Lamy, 1981; Leinert et al, 1983). However it should be noted that for micron-sized dust grains this is not true: erosion results in a rapid decrease of the particle radius with concomitant reduction of the Poynting-Robertson lifetime. Eventually the radius is sufficiently small and the solar radiation pressure sufficiently large to eject the particle from the solar system on an hyperbolic orbit (Kapisinsky, 1983).

In the next section I briefly review the present situation as regards the collisional lifetimes of meteoroidal bodies. Since a complete understanding is still some way off, a detailed discussion is not possible: only an order of magnitude estimate of the collisional lifetime is necessary. In section 4.6.2 I show how the techniques described in chapter 2 can be applied to this problem to achieve more realistic calculations than have been hitherto possible.

Since the intricacies of this problem (involving such things as the physical strength of meteoroids) are not yet understood, my only intention is to show that the technique gives results which confirm previous estimates whilst accommodating the effects of particular orbital parameters.

4.6.2 Previous Results

There have been a number of attacks upon the problem of inter-particle collisions and the lifetime against loss by such an event (τ_z: subscript z since the majority of collisions are with zodiacal cloud particles). Whipple (1967) was concerned with the maintenance of the entire meteoritic complex, and compared the lifetimes for collisions, erosions, and loss via the Poynting-Robertson mechanism. Dohnanyi (1967; 1969; 1970; 1978; and other papers) has developed models for the collisional evolution of bodies ranging in size from meteoroids to asteroids. The mass distribution index measured for various classes of objects, and its theoretical value for fragmenting or accreting systems, has been the subject of some study (for example, Napier and Dodd, 1974; Daniels and Hughes, 1981; Bishop and Searle, 1983).

In attempting to explain the mass distribution index of meteoroids and their fragmentation products, and hence the supply of particles to the zodiacal cloud (10 - 100 μm), Dohnanyi (1970; 1978) developed a very simple model. He defined τ_z as the reciprocal of the integral of the product of the encounter velocity, the collision cross-section and the spatial density over all masses capable of producing a catastrophic collision:

$$\tau_z(m) = 1/\int_{m/\Gamma}^{M_\infty} v\sigma_c\ S(M)\ dM \qquad (89)$$

where S is the spatial density of particles of mass M;

σ_c is the collision cross-section (for spherical particles, π times the sum of the squares of their radii); and

Γ is a factor which defines the minimum size of incident projectile which will cause fragmentation.

Dohnanyi adopted limits of Γ = 50 and 1000, and further simplified the calculation by assuming a constant impact velocity of 20 kms^{-1}. His results for meteoroidal particles (i.e. radius 100 μm - 1 cm) in circular orbits at 1 A.U. were (Dohnanyi 1978; Figure 24) time-scales of the order:

$\tau_z \simeq 1 \times 10^4$ years (100 μm)

$\tau_z \simeq 3 \times 10^3$ years (1 mm)

$\tau_z \simeq 1 \times 10^5$ years (1 cm)

The minimum value of τ_z for all sizes of interplanetary particle occurs at 1 mm. This result was also found by Leinert et al (1983), and is caused by the opposing influences upon τ_z of increasing meteoroid cross-section and decreasing population of dust particles capable of causing fragmentation.

A much more sophisticated model has been developed by Leinert et al (1983), whereby they take into account the particle's orbital semi-major axis and eccentricity. Using a size distribution based upon a model of Giese and Grün (1976), they find at 1 A.U. (Leinert et al, 1983; Figure 10):

$\tau_z \simeq 2 \times 10^5$ years (s $\simeq$ 100 μm)

$\tau_z \simeq 4 \times 10^4$ years (s $\simeq$ 1 mm)

$\tau_z \simeq 8 \times 10^5$ years (s $\simeq$ 1 cm)

These are about an order of magnitude longer than the values derived by Dohnanyi (1978), but still shorter than the Poynting-Robertson lifetime, τ_{PR}.

In view of the discrepancies between published collisional lifetimes, the only justifiable conclusion is that τ_z is less than $\sim 8 \times 10^5$ years for 1 cm meteoroids, and $\sim 2 \times 10^5$ years at 100 μm, both for orbits close to that of the earth. Between these size limits τ_z is somewhat less. Except for meteoroids whose initial orbits give them very small perihelion distances (e.g. the Geminids or Arietids), τ_z is of the order of 10% of τ_{PR} so that the major loss mechanism consists of collisions with the zodiacal cloud.

4.6.3 The Collisional Lifetime by the Present Method

In view of the preceding sections it is apparent that there is much room for improvement in the modelling of particle collisions in the inner solar system. In this section I will briefly show how the collision frequency method described in chapter 2 can easily be applied to meteoroid fragmentation. This method is in advance of the most refined treatment to date, that of Leinert et al (1983), in that it does not assume a spherically symmetric distribution of meteoroids and zodiacal dust particles: the influence of the various particle inclinations is easily accommodated.

Since this is purely an example of the efficacy of the technique, I shall choose as a test particle a meteoroid of radius 1 mm. Simulation experiments reported by Leinert et al (1983) indicate that for an impinging particle velocity of 10 km s^{-1}, the minimum particle radius which will cause a catastrophic collision is described by $\Gamma^3 \simeq 5 \times 10^4$, or $\Gamma \simeq 30$ to 40 (c.f. equation 89). Therefore at this velocity the minimum particle size is ~ 30 μm, for the complete fragmentation of a 1 mm meteoroid.

The models of Giese and Grün (1976) show that the spatial density of particles larger than ~ 30 μm is about 1.5×10^{-17} cm^{-3}, which I shall adopt as the standard spatial density characteristic of the environment at 1 A.U. and close to the ecliptic. This figure is in line with the mass density estimates of ~ 1×10^{-22} gm cm^{-3} by Hughes (1975), Leinert (1975), and Le Sergeant D'Hendecourt and

Lamy (1980). Denoting the spatial density of the zodiacal cloud as S_z this is therefore:

$$S_z \approx 5 \times 10^{22} \text{ (particles of } s > 30\ \mu\text{m)(A.U.)}^{-3}$$

in the region of the Earth.

At any particular solar distance R and ecliptic latitude β, S_z will be somewhat different. Spacecraft measurements have shown that S_z varies as $R^{-1.3}$ (Leinert et al, 1983), although there are local enhancements (e.g. close to the orbital distances of any of the planets or in the asteroid belt). The zodiacal cloud has a fan-shaped structure of mean inclination close to 30° (Weinberg and Sparrow, 1978; Leinert et al, 1976; 1983). The plane of symmetry is not coincidental with the ecliptic plane but is 3° away; neither is it coincidental with the invariable plane of the planets (Misconi and Weinberg, 1978; Leinert et al, 1980). Despite this small deviation I will assume that the spatial density varies as $(1 - \sin\beta)$. Thus at any radial distance R (in A.U.) and latitude β, the spatial density of particles larger than 30 μm is taken to be:

$$S_z = 5 \times 10^{22}\ (1 - \sin\beta)/R^{1.3} \qquad (\text{A.U.})^{-3} \tag{90}$$

This can then be used in equation (7), with the other spatial density found from the orbital parameters of the meteoroid in question, as described in chapter 2.

The relative velocity in the collision is calculated by assuming that the zodiacal light particle is following a circular orbit of radius R and inclination 30°, the mean for the zodiacal cloud. The resultant particle velocity is then compounded with the meteoroid velocity at the same position. The encounter velocity derived in this way is a better measure than the constant value of 20 km s^{-1} used by Dohnanyi (1978), or the pure Keplerian orbital velocity used by Leinert et al (1983). By the method used here a higher collision velocity, and hence a shorter collisional lifetime, will result for meteoroids in highly eccentric or retrograde orbits.

The final parameter required for the evaluation of equation (7) is the collision cross-section, which is taken to be $\sigma_c = \pi s^2$ with s = 1 mm. Since the majority of impacting particles have radii much smaller than the meteoroid, their finite cross-section is ignored.

The collisional lifetime can now be calculated as the reciprocal of the collision probability given by (7). For several fictitious meteor orbits and also the orbits of a few of the streams listed by Cook (1973), the characteristic lifetimes are given in Table 1.

The results are as could be anticipated. For the meteors in low eccentricity orbits (a = 1, e = 0.1) τ_z is large since the meteor never enters a region of high spatial density. As the inclination increases, so does the encounter velocity in this simple model and the lifetime is smallest for the retrograde meteor (i = 150°). The eccentric orbits

TABLE 1: COLLISIONAL LIFETIMES OF 1 mm METEOROIDS IN VARIOUS ORBITS

Name	a=(A.U.)	e=	q=(A.U.)	i=(degrees)	τ_z=(x10^3 years)	τ_z = (x10^3 orbits)
-	1.0	0.1	0.9	0	43	
-	1.0	0.1	0.9	10	44	
-	1.0	0.1	0.9	30	61	
-	1.0	0.1	0.9	90	16	
-	1.0	0.1	0.9	150	12	
-	1.0	0.9	0.1	30	13	
-	1.0	0.95	0.05	30	10	
Daytime Arietids	1.6	0.94	0.10	21.0	23	11
Geminids	1.36	0.896	0.14	23.6	22	14
Monocerotids	42	0.997	0.14	24.8	2500	9
Virginids	2.63	0.90	0.26	3.0	73	17
Halleyids	15	0.96	0.60	164	640	11
Andromedids	3.53	0.76	0.85	13.0	190	29
Ursids	5.7	0.85	0.86	53.6	330	24
Pegasids	3.86	0.75	0.97	8.0	250	33
Perseids	28	0.965	0.98	113.8	1190	8
Quadrantids	3.08	0.683	0.98	72.5	120	23
Camelopardalids	1.534	0.352	0.99	8.2	80	42

(a = 1; e = 0.9, 0.95) have decreasing lifetimes since they enter regions of high spatial density (low perihelion distance, q).

The stream orbits are arranged in order of q. Considering τ_z in units of the meteoroid orbital period (far right column), a general increase in τ_z with q is apparent, as expected. Exceptions are the Halleyids and Perseids, the retrograde paths of which limit τ_z.

From these sample orbits it is clear that for semi-major axes of 1 to 2 A.U. the collisional lifetimes range from 2 to 8 x 10^4 years, given the input model parameters. This is in line with the estimate of Leinert et al (1983) but is an order of magnitude longer than that of Dohnanyi (1978) for 1 mm meteoroids.

The above treatment was intended to illustrate the usefulness of the technique developed in Chapter 2 for calculating collision probabilities. The process can easily be repeated for 100 μm and 1 cm meteoroids with an appropriate choice of the spatial density of interplanetary particles capable of fragmenting such meteoroids. Even simpler is a scaling of the values of τ_z in Table 1 by a factor which incorporates the σ_c and S_z applicable to other meteoroid sizes. Similarly a more realistic model for the orbital distribution of zodiacal cloud particles could be used, and is an intended future extension of this work.

4.7 SUMMARY

In sections 4.3 - 4.6 the various influences which limit the lifetimes of meteoroids have been reviewed. It has been seen that the lifetimes for loss due to the Poynting-Robertson and Yarkovsky-Radzievskii effects, other radiative causes, rotational bursting, and electromagnetic influences, are longer for the size range 100 μm - 1 cm than the collisional lifetimes with zodiacal cloud particles. An exception to this could be a tiny meteoroid with a small perihelion distance (e.g. a 100 μm Geminid meteor) for which the Poynting-Robertson drag is probably the limiting factor: however, a catastrophic impact with another particle would still be its eventual fate as its orbit decayed. In view of the dominance of the collisional lifetime τ_z, the difference in definition of the other characteristic lifetimes (some being orbital decay times, others being probabilistic loss times against stochastic events) is unimportant. The results obtained here in section 4.6.3 and also by Leinert et al (1983) indicate that the following are the approximate limiting lifetimes of meteoroids:

$$\tau_z \approx 2 \times 10^5 \ p \qquad (s = 100\ \mu m)$$

$$\tau_z \approx 4 \times 10^4 \ p \qquad (s = 1\ mm) \qquad (91)$$

$$\tau_z \approx 8 \times 10^5 \ p \qquad (s = 1\ cm)$$

where p is the orbital period in years. I shall adopt these as being the time-spans against which other events are to be gauged. The events in question are sporadic meteor production by planetary disruption (τ_A, equation 83 or τ_S, equation 84) or loss by planetary collision or ejection (τ_L, equation 81).

CHAPTER 5

PLANETARY DISRUPTION OF METEOROID ORBITS

5.1 INTRODUCTION

The method discussed in chapter 2 allows the encounter probability for two objects in arbitrary Keplerian orbits to be calculated, and the techniques of chapter 3 enable the outcome of such an encounter to be investigated. One of the most general (and hence computer-time-consuming) versions of a set of programs encompassing these techniques has been included as Appendix 1 of this thesis.

The present context covers the effect of close encounters between a meteoroid (of insignificant mass and radius but orbit to be stipulated) and any of the planets. The orbits and physical parameters of the planets are therefore required; the values I have used are listed in Table 2. Pluto has been excluded since its mass and radius are now known to be insubstantial (Duncombe and Seidelmann, 1980; see also chapter 7). The radii used are those to the solid surface at the equator for the terrestrial planets, and to the visible cloud tops for the giant planets, and so represent an underestimate of the impact cross-section for a meteor.

A number of test particles are required to carry out this investigation of the gross effects of stream disruption by the planets. The test particles need to have orbital

TABLE 2: ORBITS, MASSES AND RADII OF THE PLANETS

	a=(A.U.)	e=	i=(degrees)	Mass(kg)	Radius(km)	References*
Mercury	0.3871	0.2056	7.004	3.302×10^{23}	2439	Strom (1979)
Venus	0.7233	0.0068	3.393	4.870×10^{24}	6051	Hunten et al (1977)
Earth	1.000	0.0167	0.000	5.977×10^{24}	6378	Astronomical Almanac (1984)
Mars	1.524	0.0933	1.850	6.418×10^{23}	3397	Astronomical Almanac (1984)
Jupiter	5.203	0.0484	1.305	1.899×10^{27}	71398	Astronomical Almanac (1984)
Saturn	9.539	0.0557	2.486	5.684×10^{26}	60330	Stone and Miner (1980) Anderson et al (1980)
Uranus	19.18	0.0472	0.771	8.727×10^{25}	26145	Klepczynski et al (1971) Elliot et al (1981)
Neptune	30.06	0.0086	1.776	1.030×10^{26}	24700	Klepczynski et al (1971) Freeman and Lyngå (1970)

*References apply to the masses and radii. The orbital parameters used are the mean elements from the Explanatory Supplement to the Astronomical Ephemeris (1961: pp113-115).

elements similar to those of stream meteoroids. Surveys of sporadic radio meteors at Jodrell Bank and Kharkov have been summarised by Lebedinets (1968) and also by Hughes (1978; Figure 19). These surveys include many thousands of meteors brighter than +7 radio magnitude (radius $\gtrsim$ 1 mm) with minor streams as well as sporadics being represented. Of course only meteoroids which cross the Earth's orbit are sampled. The majority of orbits are of high eccentricity ($e > 0.7$) and small perihelion distance ($q < 0.5$ A.U.) with about 60% being prograde ($i < 90^\circ$) and 40% retrograde. Most semi-major axes are between 1 and 5 A.U. but there are an appreciable number which cross Jupiter.

With consideration of the above I selected as test orbits several of the streams listed by Cook (1973). This selection was carried out with the intention of covering, as far as possible, the full range of measured meteor orbits without resorting to any 'invented' orbits. Although many of the streams of Cook (1973) have yet to be unequivocally confirmed, their use here as characteristic test particles gives a valid indication of the dynamical evolution of the meteoric complex.

The 12 streams used in Table 1 (the Halleyids comprising two streams, the Eta Aquarids and the Orionids) along with 16 other streams listed by Cook (1973) were used. The majority of these were listed as major meteor showers by Hughes (1978; Table 2) and also described by Lovell (1954) and McKinley (1961).

5.2 RESULTS

Using 28 stream orbits from Cook (1973) as test particles, the probabilities of planetary collision, deflection and ejection were calculated using equation (7) with the relevant cross-sections (chapter 3). Also found were the minimum and maximum relative velocities in all possible encounters, the approximate rate of orbital energy change (equation 78), and the mean and root-mean-square angular deflections in an encounter (equations 51 and 52). The results, using the program given in Appendix 1, are listed in Table 3 which covers the next sixteen pages. The streams are arranged in order of increasing semi-major axis, except where there is a special reason to keep two streams together on the same page (e.g. Northern and Southern Taurids).

It must be re-iterated here that the results do not apply explicitly to the streams in question with any precision. The technique used assumes that the argument of perihelion is random, which will not be true for a meteor stream unless precession and secular perturbations alter the argument of perihelion on a time-scale shorter than the stream lifetime. The intention is not to investigate the evolutionary changes of the actual streams, only the effects of close encounters upon test particles with orbits of size, shape and inclination (a,e,i) similar to each of the streams.

TABLE 3

THE EFFECTS OF CLOSE PLANETARY ENCOUNTERS UPON TEST ORBITS

For each test particle, associated with the name of a meteor stream, the following information is given:

Semi-major axis, eccentricity, inclination, period	:	from Cook (1973)
Probability (per year) of collision, deflection, or ejection (P_C, P_D, P_E)	:	for use in equations 80 and 81 to find $\tau_C, \tau_D, \tau_E, \tau_L$
Minimum and maximum encounter velocities.		
(Minus) Fractional orbital energy change per year	:	for use in equation 84 to find τ_S
Mean and root-mean square deflections in degrees at a relative velocity mid-way between the minimum and maximum.		
The total for each of P_C, P_D and P_E over all of the planets.		

Object name: GEMINIDS

Semi-major axis = 1.36 A.U.
Eccentricity = 0.8960
Inclination = 23.60 degrees
Period = 1.59 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	1.01E-09	1.86E-09	1.16E-09	1.46E-10	-	-	-	-
Deflection:	2.21E-05	1.34E-04	1.58E-04	3.93E-05	-	-	-	-
Ejection:	-	-	-	-	-	-	-	-
Velocity(km/s) Minimum:	40.9	40.1	33.5	22.2	-	-	-	-
Maximum:	65.1	41.4	34.8	28.9	-	-	-	-
Fract.Energy Change(/yr):	-7.28E-13	-1.36E-10	-2.03E-10	-2.89E-12	-	-	-	-
Deflection, Mean (deg.):	0.005	0.027	0.031	0.008	-	-	-	-
RMS (deg.):	0.008	0.045	0.054	0.015	-	-	-	-

```
*********************************************************
* Total Collision Probability =  4.18E-09 per year *
*   "   Deflection    "   "   = 3.54E-04   "    "  *
*   "   Ejection      "   "   = 0.00E+00   "    "  *
*********************************************************
```

Object name: CAMELOPARDALIDS

Semi-major axis = 1.534 A.U.
Eccentricity = 0.3520
Inclination = 8.20 degrees
Period = 1.90 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	1.52E-08	3.79E-10	-	-	-	-
Deflection:	-	-	6.56E-04	8.08E-05	-	-	-	-
Ejection:	-	-	-	-	-	-	-	-
Velocity(km/s) Minimum:	-	-	6.7	6.8	-	-	-	-
Maximum:	-	-	7.5	11.8	-	-	-	-
Fract.Energy Change(/yr):	-	-	+1.24E-07	-4.30E-11	-	-	-	-
Deflection, Mean (deg.):	-	-	0.730	0.063	-	-	-	-
RMS (deg.):	-	-	1.184	0.110	-	-	-	-

```
*********************************************************
* Total Collision Probability =  1.55E-08 per year *
*   "   Deflection    "   "   = 7.37E-04   "    "  *
*   "   Ejection      "   "   = 0.00E+00   "    "  *
*********************************************************
```

Object name: DAYTIME ZETA PERSEIDS

Semi-major axis	=	1.60 A.U.
Eccentricity	=	0.7900
Inclination	=	0.00 degrees
Period	=	2.02 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	2.28E-09	9.38E-09	6.85E-07	1.26E-09	-	-	-	-
Deflection:	5.27E-05	6.43E-04	8.84E-02	3.34E-04	-	-	-	-
Ejection:	-	-	-	-	-	-	-	-
Velocity(km/s) Minimum:	16.1	29.6	26.5	18.8	-	-	-	-
Maximum:	36.2	30.1	27.6	24.3	-	-	-	-
Fract.Energy Change (/yr):	+3.10E-11	-5.36E-10	-1.44E-07	-3.45E-11	-	-	-	-
Deflection, Mean (deg.):	0.020	0.050	0.050	0.012	-	-	-	-
RMS (deg.):	0.032	0.083	0.086	0.021	-	-	-	-

```
*******************************************************
* Total Collision Probability =  6.98E-07 per year *
*    "   Deflection    "   "  =  8.94E-02  "    "  *
*    "   Ejection      "   "  =  0.00E+00  "    "  *
*******************************************************
```

Object name: DAYTIME ARIETIDS

Semi-major axis	=	1.60 A.U.
Eccentricity	=	0.9400
Inclination	=	21.00 degrees
Period	=	2.02 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	8.82E-10	1.62E-09	1.00E-09	1.24E-10	-	-	-	-
Deflection:	1.93E-05	1.18E-04	1.39E-04	3.36E-05	-	-	-	-
Ejection:	-	-	-	-	-	-	-	-
Velocity(km/s) Minimum:	46.6	43.7	36.7	25.4	-	-	-	-
Maximum:	71.1	44.9	37.9	31.9	-	-	-	-
Fract.Energy Change (/yr):	-7.26E-13	-1.20E-10	-1.67E-10	-2.02E-12	-	-	-	-
Deflection, Mean (deg.):	0.004	0.023	0.026	0.007	-	-	-	-
RMS (deg.):	0.006	0.038	0.046	0.012	-	-	-	-

```
*******************************************************
* Total Collision Probability =  3.63E-09 per year *
*    "   Deflection    "   "  =  3.10E-04  "    "  *
*    "   Ejection      "   "  =  0.00E+00  "    "  *
*******************************************************
```

Object name: NORTHERN IOTA AQUARIDS

Semi-major axis = 1.75 A.U.
Eccentricity = 0.8400
Inclination = 5.00 degrees
Period = 2.32 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	2.68E-09	6.43E-09	3.42E-09	4.22E-10	-	-	-	-
Deflection:	5.73E-05	4.50E-04	4.54E-04	1.13E-04	-	-	-	-
Ejection:	-	-	-	-	-	-	-	-
Velocity(km/s) Minimum:	20.4	33.0	29.2	21.0	-	-	-	-
Maximum:	44.1	33.8	30.3	26.7	-	-	-	-
Fract.Energy Change(/yr):	+1.12E-11	-4.72E-10	-7.02E-10	-9.60E-12	-	-	-	-
Deflection, Mean (deg.):	0.013	0.040	0.041	0.010	-	-	-	-
RMS (deg.):	0.021	0.067	0.071	0.017	-	-	-	-

```
*****************************************************
* Total Collision Probability = 1.30E-08 per year *
*   "   Deflection    "    "   = 1.07E-03  "   "  *
*   "   Ejection      "    "   = 0.00E+00  "   "  *
*****************************************************
```

Object name: SOUTHERN IOTA AQUARIDS

Semi-major axis = 2.36 A.U.
Eccentricity = 0.9120
Inclination = 6.90 degrees
Period = 3.63 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	2.67E-09	2.71E-09	1.56E-09	1.90E-10	-	-	-	-
Deflection:	5.83E-05	1.95E-04	2.13E-04	5.13E-05	-	-	-	-
Ejection:	-	-	-	-	-	-	-	-
Velocity(km/s) Minimum:	32.9	38.4	33.7	25.2	-	-	-	-
Maximum:	54.4	39.1	34.9	30.9	-	-	-	-
Fract.Energy Change(/yr):	+5.56E-14	-2.51E-10	-3.33E-10	-3.63E-12	-	-	-	-
Deflection, Mean (deg.):	0.007	0.029	0.031	0.007	-	-	-	-
RMS (deg.):	0.011	0.049	0.054	0.012	-	-	-	-

```
*****************************************************
* Total Collision Probability = 7.14E-09 per year *
*   "   Deflection    "    "   = 5.17E-04  "   "  *
*   "   Ejection      "    "   = 0.00E+00  "   "  *
*****************************************************
```

Object name: DAYTIME BETA TAURIDS

Semi-major axis = 2.20 A.U.
Eccentricity = 0.8500
Inclination = 6.00 degrees
Period = 3.26 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	1.98E-09	3.58E-09	2.00E-09	2.39E-10	-	-	-	-
Deflection:	4.49E-05	2.49E-04	2.64E-04	6.40E-05	-	-	-	-
Ejection:	-	-	-	-	-	-	-	-
Velocity(km/s) Minimum:	15.3	31.8	29.2	22.1	-	-	-	-
Maximum:	39.3	32.7	30.2	27.5	-	-	-	-
Fract.Energy Change (/yr):	+3.31E-11	-2.07E-10	-4.08E-10	-5.19E-12	-	-	-	-
Deflection, Mean (deg.):	0.018	0.043	0.042	0.009	-	-	-	-
RMS (deg.):	0.029	0.071	0.072	0.016	-	-	-	-

```
*****************************************************
* Total Collision Probability =  7.80E-09 per year *
*   "   Deflection   "    "   =  6.23E-04   "    "  *
*   "   Ejection     "    "   =  0.00E+00   "    "  *
*****************************************************
```

Object name: ALPHA CAPRICORNIDS

Semi-major axis = 2.53 A.U.
Eccentricity = 0.7700
Inclination = 7.00 degrees
Period = 4.02 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	2.97E-09	1.48E-09	1.60E-10	-	-	-	-
Deflection:	-	1.82E-04	1.81E-04	4.23E-05	-	-	-	-
Ejection:	-	-	-	-	-	-	-	-
Velocity(km/s) Minimum:	-	20.0	22.4	18.7	-	-	-	-
Maximum:	-	21.4	23.4	23.6	-	-	-	-
Fract.Energy Change (/yr):	-	+2.09E-09	+2.94E-11	-3.69E-12	-	-	-	-
Deflection, Mean (deg.):	-	0.103	0.070	0.012	-	-	-	-
RMS (deg.):	-	0.172	0.120	0.022	-	-	-	-

```
*****************************************************
* Total Collision Probability =  4.62E-09 per year *
*   "   Deflection   "    "   =  4.05E-04   "    "  *
*   "   Ejection     "    "   =  0.00E+00   "    "  *
*****************************************************
```

Object name: NORTHERN TAURIDS

Semi-major axis = 2.59 A.U.
Eccentricity = 0.8610
Inclination = 2.40 degrees
Period = 4.17 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	9.98E-10	5.35E-09	3.87E-09	5.61E-10	-	-	-	-
Deflection:	2.48E-05	3.70E-04	5.11E-04	1.51E-04	-	-	-	-
Ejection:	-	-	-	-	-	-	-	-
Velocity (km/s) Minimum:	17.5	31.0	29.0	22.6	-	-	-	-
Maximum:	35.4	31.7	30.0	27.8	-	-	-	-
Fract.Energy Change (/yr):	+2.69E-11	-1.84E-10	-7.79E-10	-1.22E-11	-	-	-	-
Deflection, Mean (deg.):	0.020	0.045	0.042	0.009	-	-	-	-
RMS (deg.):	0.031	0.075	0.073	0.015	-	-	-	-

```
**********************************************************
* Total Collision Probability =  1.08E-08 per year *
*   "   Deflection    "    "  =  1.06E-03  "   "   *
*   "   Ejection      "    "  =  0.00E+00  "   "   *
**********************************************************
```

Object name: SOUTHERN TAURIDS

Semi-major axis = 1.93 A.U.
Eccentricity = 0.8060
Inclination = 5.20 degrees
Period = 2.68 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	1.69E-09	5.33E-09	2.85E-09	3.40E-10	-	-	-	-
Deflection:	4.37E-05	3.64E-04	3.69E-04	9.05E-05	-	-	-	-
Ejection:	-	-	-	-	-	-	-	-
Velocity (km/s) Minimum:	16.6	28.9	26.9	20.1	-	-	-	-
Maximum:	33.4	29.8	27.9	25.5	-	-	-	-
Fract.Energy Change (/yr):	+4.28E-11	-1.69E-10	-5.59E-10	-8.22E-12	-	-	-	-
Deflection, Mean (deg.):	0.022	0.051	0.049	0.011	-	-	-	-
RMS (deg.):	0.035	0.086	0.084	0.019	-	-	-	-

```
**********************************************************
* Total Collision Probability =  1.02E-08 per year *
*   "   Deflection    "    "  =  8.67E-04  "   "   *
*   "   Ejection      "    "  =  0.00E+00  "   "   *
**********************************************************
```

Object name: NORTHERN DELTA AQUARIDS

Semi-major axis = 2.62 A.U.
Eccentricity = 0.9700
Inclination = 20.00 degrees
Period = 4.24 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	4.42E-10	8.03E-10	4.86E-10	5.90E-11	1.19E-07	-	-	-
Deflection:	9.66E-06	5.89E-05	6.84E-05	1.61E-05	6.22E-03	-	-	-
Ejection:	-	-	-	-	4.71E-07	-	-	-
Velocity(km/s) Minimum:	50.5	47.2	40.2	29.6	11.1	-	-	-
Maximum:	75.1	48.2	41.4	35.8	12.1	-	-	-
Fract.Energy Change(/yr):	-5.30E-13	-7.84E-11	-9.95E-11	-9.74E-13	0.00E+00	-	-	-
Deflection, Mean (deg.):	0.003	0.019	0.022	0.005	2.428	-	-	-
RMS (deg.):	0.006	0.033	0.038	0.009	3.744	-	-	-

```
*********************************************************
* Total Collision Probability =  1.21E-07 per year *
*   "    Deflection    "    "  = 6.38E-03  "    "  *
*   "    Ejection      "    "  = 4.71E-07  "    "  *
*********************************************************
```

Object name: SOUTHERN DELTA AQUARIDS

Semi-major axis = 2.86 A.U.
Eccentricity = 0.9760
Inclination = 27.20 degrees
Period = 4.84 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	2.92E-10	5.32E-10	3.22E-10	3.89E-11	7.72E-08	-	-	-
Deflection:	6.38E-06	3.91E-05	4.55E-05	1.06E-05	4.75E-03	-	-	-
Ejection:	-	-	-	-	3.88E-07	-	-	-
Velocity(km/s) Minimum:	52.8	48.7	41.4	30.6	11.2	-	-	-
Maximum:	78.4	49.8	42.7	36.8	13.6	-	-	-
Fract.Energy Change(/yr):	-3.74E-13	-5.44E-11	-6.83E-11	-6.51E-13	-1.51E-05	-	-	-
Deflection, Mean (deg.):	0.003	0.018	0.021	0.005	2.139	-	-	-
RMS (deg.):	0.005	0.031	0.036	0.008	3.334	-	-	-

```
*********************************************************
* Total Collision Probability =  7.84E-08 per year *
*   "    Deflection    "    "  = 4.85E-03  "    "  *
*   "    Ejection      "    "  = 3.88E-07  "    "  *
*********************************************************
```

Object name: VIRGINIDS

Semi-major axis = 2.63 A.U.
Eccentricity = 0.9000
Inclination = 3.00 degrees
Period = 4.27 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	1.25E-09	6.48E-09	3.01E-09	4.01E-10	1.01E-06	-	-	-
Deflection:	2.69E-05	4.60E-04	4.07E-04	1.08E-04	3.49E-02	-	-	-
Ejection:	-	-	-	-	3.39E-06	-	-	-
Velocity (km/s) Minimum:	25.1	35.9	32.2	24.6	9.4	-	-	-
Maximum:	47.2	36.5	33.3	30.1	9.7	-	-	-
Fract.Energy Change (/yr):	+4.39E-12	-5.61E-10	-6.69E-10	-8.17E-12	0.00E+00	-	-	-
Deflection, Mean (deg.):	0.010	0.034	0.034	0.007	3.536	-	-	-
RMS (deg.):	0.017	0.057	0.059	0.013	5.271	-	-	-

```
**********************************************************
* Total Collision Probability =  1.02E-06 per year *
*   "  Deflection    "    "   = 3.59E-02  "    "   *
*   "  Ejection      "    "   = 3.39E-06  "    "   *
**********************************************************
```

Object name: THETA OPHIUCHIDS

Semi-major axis = 2.90 A.U.
Eccentricity = 0.8400
Inclination = 4.00 degrees
Period = 4.94 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	5.84E-10	4.64E-09	1.99E-09	2.37E-10	9.32E-07	-	-	-
Deflection:	1.83E-05	3.10E-04	2.56E-04	6.33E-05	2.69E-02	-	-	-
Ejection:	-	-	-	-	3.65E-06	-	-	-
Velocity (km/s) Minimum:	20.4	26.4	26.3	21.5	7.6	-	-	-
Maximum:	22.6	27.3	27.3	26.5	9.4	-	-	-
Fract.Energy Change (/yr):	0.00E+00	+6.55E-10	-2.71E-10	-5.30E-12	-2.36E-04	-	-	-
Deflection, Mean (deg.):	0.030	0.061	0.051	0.009	4.460	-	-	-
RMS (deg.):	0.047	0.103	0.088	0.017	6.501	-	-	-

```
**********************************************************
* Total Collision Probability =  9.40E-07 per year *
*   "  Deflection    "    "   = 2.76E-02  "    "   *
*   "  Ejection      "    "   = 3.65E-06  "    "   *
**********************************************************
```

Object name: DECEMBER PHOENICIDS BRANCH A

Semi-major axis = 2.96 A.U.
Eccentricity = 0.6800
Inclination = 16.00 degrees
Period = 5.09 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	1.24E-09	6.00E-11	2.00E-07	-	-	-
Deflection:	-	-	1.17E-04	1.54E-05	3.53E-03	-	-	-
Ejection:	-	-	3.74E-12	-	3.08E-07	-	-	-
Velocity(km/s) Minimum:	-	-	14.0	15.0	6.7	-	-	-
Maximum:	-	-	15.0	19.8	6.9	-	-	-
Fract.Energy Change(/yr):	-	-	+3.93E-09	-3.31E-13	0.00E+00	-	-	-
Deflection, Mean (deg.):	-	-	0.175	0.018	7.008	-	-	-
RMS (deg.):	-	-	0.296	0.032	9.749	-	-	-

```
*********************************************************
* Total Collision Probability =  2.02E-07 per year *
*   "   Deflection     "    "  = 3.67E-03  "    "   *
*   "   Ejection       "    "  = 3.08E-07  "    "   *
*********************************************************
```

Object name: DECEMBER PHOENICIDS BRANCH B

Semi-major axis = 2.96 A.U.
Eccentricity = 0.6700
Inclination = 13.00 degrees
Period = 5.09 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	2.98E-09	7.27E-11	1.47E-07	-	-	-
Deflection:	-	-	2.41E-04	1.85E-05	2.36E-03	-	-	-
Ejection:	-	-	-	-	1.31E-07	-	-	-
Velocity(km/s) Minimum:	-	-	11.7	14.1	6.4	-	-	-
Maximum:	-	-	13.0	18.9	6.6	-	-	-
Fract.Energy Change(/yr):	-	-	+1.76E-08	+2.26E-13	0.00E+00	-	-	-
Deflection, Mean (deg.):	-	-	0.241	0.020	7.643	-	-	-
RMS (deg.):	-	-	0.406	0.035	10.533	-	-	-

```
*********************************************************
* Total Collision Probability =  1.50E-07 per year *
*   "   Deflection     "    "  = 2.62E-03  "    "   *
*   "   Ejection       "    "  = 1.31E-07  "    "   *
*********************************************************
```

Object name: QUADRANTIDS

Semi-major axis = 3.08 A.U.
Eccentricity = 0.6830
Inclination = 72.50 degrees
Period = 5.41 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	1.13E-09	2.99E-11	4.59E-08	-	-	-
Deflection:	-	-	1.58E-04	8.10E-06	3.19E-03	-	-	-
Ejection:	-	-	-	-	2.74E-07	-	-	-
Velocity(km/s) Minimum:	-	-	40.7	30.2	12.8	-	-	-
Maximum:	-	-	41.6	36.3	14.2	-	-	-
Fract.Energy Change (/yr):	-	-	-2.56E-10	-5.30E-13	0.00E+00	-	-	-
Deflection, Mean (deg.):	-	-	0.022	0.005	1.794	-	-	-
RMS (deg.):	-	-	0.037	0.009	2.838	-	-	-

```
*****************************************************
* Total Collision Probability = 4.70E-08 per year *
*   "   Deflection    "   "   = 3.36E-03  "    "  *
*   "   Ejection      "   "   = 2.74E-07  "    "  *
*****************************************************
```

Object name: KAPPA CYGNIDS

Semi-major axis = 3.09 A.U.
Eccentricity = 0.6800
Inclination = 38.00 degrees
Period = 5.43 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	1.18E-09	3.24E-11	9.35E-08	-	-	-
Deflection:	-	-	1.47E-04	8.56E-06	3.03E-03	-	-	-
Ejection:	-	-	-	-	4.64E-07	-	-	-
Velocity(km/s) Minimum:	-	-	23.8	20.1	8.4	-	-	-
Maximum:	-	-	24.0	24.8	9.8	-	-	-
Fract.Energy Change (/yr):	-	-	+5.17E-11	-6.95E-13	0.00E+00	-	-	-
Deflection, Mean (deg.):	-	-	0.064	0.011	3.913	-	-	-
RMS (deg.):	-	-	0.110	0.019	5.777	-	-	-

```
*****************************************************
* Total Collision Probability = 9.47E-08 per year *
*   "   Deflection    "   "   = 3.18E-03  "    "  *
*   "   Ejection      "   "   = 4.64E-07  "    "  *
*****************************************************
```

Object name: OCTOBER DRACONIDS

Semi-major axis = 3.51 A.U.
Eccentricity = 0.7170
Inclination = 30.70 degrees
Period = 6.58 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	9.19E-10	2.92E-11	5.51E-08	-	-	-
Deflection:	-	-	1.08E-04	7.67E-06	2.07E-03	-	-	-
Ejection:	-	-	4.86E-11	-	4.08E-07	-	-	-
Velocity(km/s) Minimum:	-	-	20.3	18.6	8.5	-	-	-
Maximum:	-	-	20.6	23.2	10.6	-	-	-
Fract.Energy Change (/yr):	-	-	+6.32E-10	-4.82E-13	-1.21E-05	-	-	-
Deflection, Mean (deg.):	-	-	0.088	0.012	3.559	-	-	-
RMS (deg.):	-	-	0.150	0.022	5.301	-	-	-

```
*****************************************************
* Total Collision Probability =  5.61E-08 per year *
*   "   Deflection    "    "  =  2.18E-03  "    "  *
*   "   Ejection      "    "  =  4.08E-07  "    "  *
*****************************************************
```

Object name: ANDROMEDIDS

Semi-major axis = 3.53 A.U.
Eccentricity = 0.7600
Inclination = 13.00 degrees
Period = 6.63 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	6.75E-10	5.33E-11	1.15E-07	-	-	-
Deflection:	-	-	7.19E-05	1.39E-05	3.92E-03	-	-	-
Ejection:	-	-	5.33E-11	-	8.33E-07	-	-	-
Velocity(km/s) Minimum:	-	-	16.8	16.9	8.1	-	-	-
Maximum:	-	-	17.8	21.7	10.1	-	-	-
Fract.Energy Change (/yr):	-	-	+1.29E-09	-4.80E-13	-2.61E-05	-	-	-
Deflection, Mean (deg.):	-	-	0.123	0.015	3.927	-	-	-
RMS (deg.):	-	-	0.209	0.026	5.795	-	-	-

```
*****************************************************
* Total Collision Probability =  1.16E-07 per year *
*   "   Deflection    "    "  =  4.01E-03  "    "  *
*   "   Ejection      "    "  =  8.33E-07  "    "  *
*****************************************************
```

Object name: PEGASIDS

Semi-major axis = 3.86 A.U.
Eccentricity = 0.7500
Inclination = 8.00 degrees
Period = 7.58 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	1.95E-09	7.58E-11	1.33E-07	-	-	-
Deflection:	-	-	1.57E-04	1.95E-05	4.93E-03	-	-	-
Ejection:	-	-	-	-	1.23E-06	-	-	-
Velocity(km/s) Minimum:	-	-	11.3	15.0	8.5	-	-	-
Maximum:	-	-	12.9	19.8	10.4	-	-	-
Fract.Energy Change (/yr):	-	-	+1.89E-08	+1.30E-12	-2.84E-05	-	-	-
Deflection, Mean (deg.):	-	-	0.250	0.018	3.627	-	-	-
RMS (deg.):	-	-	0.421	0.032	5.393	-	-	-

```
*********************************************************
* Total Collision Probability =  1.35E-07 per year *
*   "   Deflection    "    "   =  5.11E-03   "    "   *
*   "   Ejection      "    "   =  1.23E-06   "    "   *
*********************************************************
```

Object name: URSIDS

Semi-major axis = 5.70 A.U.
Eccentricity = 0.8500
Inclination = 53.60 degrees
Period = 13.61 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	1.38E-10	9.94E-12	5.15E-09	2.19E-09	-	-
Deflection:	-	-	1.88E-05	2.68E-06	5.04E-04	4.04E-04	-	-
Ejection:	-	-	1.37E-11	-	1.12E-07	7.76E-09	-	-
Velocity(km/s) Minimum:	-	-	34.1	27.3	14.7	7.8	-	-
Maximum:	-	-	34.7	32.5	16.6	10.0	-	-
Fract.Energy Change (/yr):	-	-	-4.75E-11	-2.86E-13	-1.06E-06	-3.21E-07	-	-
Deflection, Mean (deg.):	-	-	0.031	0.006	1.337	1.021	-	-
RMS (deg.):	-	-	0.054	0.011	2.164	1.695	-	-

```
*********************************************************
* Total Collision Probability =  7.48E-09 per year *
*   "   Deflection    "    "   =  9.30E-04   "    "   *
*   "   Ejection      "    "   =  1.19E-07   "    "   *
*********************************************************
```

Object name: LEONIDS

Semi-major axis = 11.50 A.U.
Eccentricity = 0.9150
Inclination = 162.60 degrees
Period = 39.00 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	7.96E-10	1.75E-11	2.69E-09	4.74E-10	3.99E-11	-
Deflection:	-	-	1.16E-04	4.78E-06	6.18E-04	2.77E-04	1.31E-04	-
Ejection:	-	-	-	-	1.15E-07	4.58E-09	6.30E-11	-
Velocity(km/s) Minimum:	-	-	69.3	49.7	24.1	15.8	8.8	-
Maximum:	-	-	71.1	58.5	26.2	17.7	10.1	-
Fract.Energy Change (/yr):	-	-	-3.13E-10	-4.98E-13	-1.05E-06	-7.60E-08	-3.71E-09	-
Deflection, Mean (deg.):	-	-	0.007	1.86E-03	0.520	0.285	0.128	-
RMS (deg.):	-	-	0.013	0.003	0.894	0.511	0.244	-

```
*********************************************************
* Total Collision Probability =  4.02E-09 per year *
*   "   Deflection    "    "   =  1.15E-03  "    "  *
*   "   Ejection      "    "   =  1.20E-07  "    "  *
*********************************************************
```

Object name: ETA AQUARIDS

Semi-major axis = 13.00 A.U.
Eccentricity = 0.9580
Inclination = 163.50 degrees
Period = 46.87 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	2.59E-10	1.07E-10	1.06E-11	2.23E-09	3.86E-10	2.41E-11	-
Deflection:	-	1.95E-05	1.58E-05	2.91E-06	4.84E-04	2.19E-04	8.11E-05	-
Ejection:	-	-	-	-	1.35E-07	5.10E-09	5.49E-11	-
Velocity(km/s) Minimum:	-	79.4	65.2	47.3	23.3	15.6	9.0	-
Maximum:	-	81.3	67.0	55.5	25.4	17.4	10.2	-
Fract.Energy Change (/yr):	-	-6.13E-11	-5.29E-11	-3.70E-13	-9.57E-07	-6.74E-08	-2.33E-09	-
Deflection, Mean (deg.):	-	0.007	0.008	0.002	0.554	0.296	0.125	-
RMS (deg.):	-	0.012	0.015	0.004	0.950	0.530	0.238	-

```
*********************************************************
* Total Collision Probability =  3.01E-09 per year *
*   "   Deflection    "   "   =  8.23E-04  "   "   *
*   "   Ejection      "   "   =  1.41E-07  "   "   *
*********************************************************
```

Object name: ORIONIDS

Semi-major axis = 15.10 A.U.
Eccentricity = 0.9620
Inclination = 163.90 degrees
Period = 58.68 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	2.32E-10	9.09E-11	8.79E-12	1.78E-09	3.00E-10	1.59E-11	1.28E-11
Deflection:	-	1.75E-05	1.34E-05	2.42E-06	3.94E-04	1.76E-04	5.83E-05	5.29E-05
Ejection:	-	-	-	-	1.41E-07	5.17E-09	5.00E-11	4.23E-11
Velocity(km/s) Minimum:	-	80.0	65.6	47.6	23.6	15.9	9.5	6.4
Maximum:	-	81.9	67.4	55.8	25.7	17.7	10.7	6.5
Fract.Energy Change (/yr):	-	-6.30E-11	-5.15E-11	-3.52E-13	-8.73E-07	-5.89E-08	-1.60E-09	0.00E+00
Deflection, Mean (deg.):	-	0.007	0.008	0.002	0.542	0.284	0.112	0.196
RMS (deg.):	-	0.011	0.014	0.004	0.930	0.510	0.215	0.369

```
*********************************************************
* Total Collision Probability =  2.44E-09 per year *
*   "   Deflection    "   "   =  7.15E-04  "   "   *
*   "   Ejection      "   "   =  1.46E-07  "   "   *
*********************************************************
```

Object name: PERSEIDS

Semi-major axis = 28.00 A.U.
Eccentricity = 0.9650
Inclination = 113.80 degrees
Period = 148.16 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	7.08E-11	1.28E-12	2.18E-10	3.31E-11	1.35E-12	1.10E-12
Deflection:	-	-	1.02E-05	3.49E-07	4.55E-05	1.94E-05	5.62E-06	6.78E-06
Ejection:	-	-	7.79E-11	-	6.45E-08	2.03E-09	1.54E-11	1.26E-11
Velocity(km/s) Minimum:	-	-	59.6	43.7	22.7	15.9	10.4	7.8
Maximum:	-	-	61.1	51.5	24.8	17.7	11.4	8.0
Fract.Energy Change (/yr):	-	-	-8.30E-11	-1.05E-13	-1.84E-07	-1.07E-08	-1.95E-10	-2.67E-10
Deflection, Mean (deg.):	-	-	0.010	0.002	0.581	0.285	0.096	0.130
RMS (deg.):	-	-	0.017	0.004	0.994	0.510	0.185	0.250

```
**********************************************************
* Total Collision Probability =  3.26E-10 per year *
*   "   Deflection     "    "   =  8.79E-05  "    "   *
*   "   Ejection       "    "   =  6.67E-08  "    "   *
**********************************************************
```

Object name: APRIL LYRIDS

Semi-major axis = 28.00 A.U.
Eccentricity = 0.9680
Inclination = 79.00 degrees
Period = 148.16 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	-	-	1.49E-11	8.99E-13	2.21E-10	3.27E-11	1.31E-12	1.08E-12
Deflection:	-	-	2.12E-06	2.45E-07	3.72E-05	1.63E-05	4.83E-06	5.85E-06
Ejection:	-	-	6.28E-11	-	8.44E-08	2.44E-09	1.72E-11	1.35E-11
Velocity(km/s) Minimum:	-	-	46.3	35.5	20.1	14.4	9.6	7.2
Maximum:	-	-	47.2	41.7	22.0	16.1	10.6	7.5
Fract.Energy Change (/yr):	-	-	-2.28E-11	-9.30E-14	-1.70E-07	-1.01E-08	-1.88E-10	-2.64E-10
Deflection, Mean (deg.):	-	-	0.017	0.004	0.741	0.345	0.112	0.151
RMS (deg.):	-	-	0.029	0.006	1.249	0.612	0.215	0.289

```
**********************************************************
* Total Collision Probability =  2.71E-10 per year *
*   "   Deflection     "    "   =  6.66E-05  "    "   *
*   "   Ejection       "    "   =  8.69E-08  "    "   *
**********************************************************
```

Object name: MONOCEROTIDS

Semi-major axis	=	42.00 A.U.
Eccentricity	=	0.9970
Inclination	=	24.80 degrees
Period	=	272.19 years

	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE
Probability (per year) of Collision:	5.55E-12	9.91E-12	5.93E-12	7.06E-13	2.57E-10	3.85E-11	1.47E-12	1.07E-12
Deflection:	1.21E-07	7.28E-07	8.39E-07	1.93E-07	4.16E-05	1.91E-05	5.67E-06	6.61E-06
Ejection:	9.97E-13	1.09E-10	7.80E-11	1.28E-13	2.17E-07	6.42E-09	4.27E-11	2.91E-11
Velocity(km/s) Minimum:	48.0	47.8	42.2	33.2	19.7	14.5	9.9	7.8
Maximum:	71.9	49.0	43.4	39.0	21.4	16.0	10.9	8.0
Fract.Energy Change(/yr):	-8.52E-14	-1.20E-11	-1.40E-11	-1.13E-13	-2.79E-07	-1.66E-08	-2.81E-10	-3.27E-10
Deflection, Mean (deg.):	0.004	0.019	0.020	0.004	0.776	0.347	0.106	0.131
RMS (deg.):	0.006	0.032	0.035	0.007	1.303	0.616	0.203	0.252

```
*******************************************************
* Total Collision Probability =  3.20E-10 per year  *
*   "    Deflection   "    "  =  7.49E-05   "    "  *
*   "    Ejection     "    "  =  2.23E-07   "    "  *
*******************************************************
```

5.3 DISCUSSION

A few pertinent remarks upon the contents of Table 3 can be made here.

(a) No ejection is possible for any of the test particles unless the orbit crosses Jupiter: this requires a semi-major axis of at least 2.5 A.U.

(b) The terrestrial planets can eject meteors from the solar system but a high initial orbital energy is required. Advancing through Table 3 the first meteoroid which can be ejected by any of the four inner planets is that in the orbit of the December Phoenicids - Branch A ($a \approx 3$ A.U.). Only a highly energetic meteor can be ejected by Mars or Mercury (e.g. a Monocerotid meteor).

(c) As soon as ejection is possible, it dominates planetary collisions. The only meteoroid to have $P_C > P_E$ (for $P_E \neq 0$) is that in the orbit of the December Phoenicids - Branch B.

(d) For several of the meteors the fractional orbital energy change is listed as zero even though an encounter is possible (e.g. Virginids and Jupiter, Theta Ophiuchids and Mercury). This is because the meteoroid's perihelion or aphelion does not cross the planet's semi-major axis, negating the method described in 3.6 as was noted at the end of that section.

(e) The deflections produced by the terrestrial planets are small, being much less than one degree except in the few cases where the encounter velocity is small (e.g. for the 'toroidal' Camelopardalids). A close approach would therefore tend to broaden a stream rather than scatter the meteors into widely different orbits.

(f) The deflections produced by Jupiter are much higher, being of the order of a degree or more. Thus it is apparent from only a cursory inspection that Jupiter, aided by the other giant planets, is responsible for such disruption of streams into sporadic meteors as occurs as a result of planetary close encounters. This is discussed in more detail in chapter 6.

(g) The energy changes deduced are mostly negative, although caution must be applied due to the rudimentary assumptions made in deriving equation 78. This clearly shows that the overall effect of close encounters is to reduce the size of the meteor orbits, as is to be expected from the work which has been done on the evolution of cometary orbits (e.g. Yabushita, 1983, for a review).

Of much more importance than the brief notes above is the following, which is one rationale for an approach using many distinct test particle orbits. Weidenschilling (1975a) and also Fernandez (1978) both used an expression equivalent to my equation (64) for the ejection probability per encounter, but with an assumed distribution of encounter angle of the form

$$\frac{\sin \epsilon}{(1-\cos\epsilon_c)}$$

i.e. all orientations of the incoming relative velocity vector are equally likely except that the loss cone ($\epsilon < \epsilon_c$) is excluded. This leads to a major overestimate by both Weidenschilling and Fernandez of the ejection probability since there are then many particles with encounter angle ϵ only slightly greater than the critical angle ϵ_c, so that a very small deflection is capable of ejecting the particle. The average ejection probability at any particular encounter is therefore greatly enhanced on account of the assumed encounter-angle distribution. Using this type of distribution they found (Weidenschilling, 1975a: Figures 4 and 5; Fernandez, 1978: Figure 3):

$$\begin{aligned} P_E/P_C &\simeq 1000 && \text{(Giant Planets)} \\ &\simeq 10 && \text{(Venus, Earth, Mars)} \\ &\simeq 1 && \text{(Mercury)} \end{aligned}$$

Both authors noted that the distribution used is unreasonable since rapid ejection of particles with ϵ close to ϵ_c would be expected, with replenishment due to gradual diffusive changes in ϵ as a result of other close encounters. The above values for (P_E/P_C) should therefore represent upper limits.

This surmise is entirely borne out by a consideration of Table 3. Except for the two branches of the December Phoenicids which have extremely low ejection probabilities, whenever ejection by Jupiter is possible, $(P_E/P_C) > 3$. This figure occurs for the Virginids (a = 2.63 A.U.), and (P_E/P_C) increases to become greater than 10 for the Ursids (a = 5.5 A.U.), and greater than 100 for the Perseids and April Lyrids (a = 28 A.U.). The ratio only approaches 1000 for the Monocerotids (a = 42 A.U.).

The above figures were all referred to Jupiter. For the Earth, whenever ejection is possible from the smaller meteor orbits (e.g. October Draconids, Andromedids, Ursids), the ratio (P_E/P_C) is of the order of 0.1. The figure becomes greater than 1 for the larger orbits (Perseids, April Lyrids), and for the Monocerotids the following are derived

$$(P_E/P_C) \simeq 0.2 \qquad \text{(Mercury, Mars)}$$
$$\simeq 12 \qquad \text{(Venus, Earth)}$$
$$\simeq 850 \qquad \text{(Jupiter)}$$
$$\simeq 170 \qquad \text{(Saturn)}$$
$$\simeq 30 \qquad \text{(Uranus, Neptune)}$$

These figures are comparable with those of Weidenschilling (1975a) for Mercury, Venus, Jupiter and the Earth, but are lower for the other four planets. However, the above are for just one test orbit, and the untenable distribution used in the published work leads to an overestimate of the importance of ejections. Therefore there is no cause to doubt the validity of the results of Table 3.

This chapter has given the results of applying the close encounter theory expostulated earlier in this thesis to a number of test particles in orbits similar to known meteor streams. In chapter 6 the appropriate time-scales based upon these results are calculated.

CHAPTER 6

COMPARISON OF THE VARIOUS METEOROID LIFETIMES

6.1 PRESENTATION OF THE DATA

Following on from the results of chapter 5, the lifetimes of interest can now be calculated for the 28 stream orbits. Using the probabilities listed in Table 3 the lifetime against loss due to planetary collision or ejection (τ_L) can be calculated via equation (81), and similarly the lifetime against any close encounter (τ_D). The time-scale for sporadic production by accumulated deflections (τ_A) is found from equation (83) using P_D and χ_{RMS}, and the rate of energy change is used to find an alternate time-scale for sporadic production (τ_S) from equation (84). Those time-scales noted so far apply to any test particle (asteroid, comet, meteoroid) and are size-independent.

Using an assumed radius of 1 mm the Poynting-Robertson lifetime (τ_{PR}) is derived from equation (87); also using this radius the lifetime against catastrophic collisions with other particles, mainly smaller zodiacal cloud grains, is calculated (τ_Z: equation 91). Reference to equations (87,91) shows that for a 100 μm meteoroid τ_{PR} is ten times shorter and τ_Z five times longer than for the assumed 1 mm meteoroid. Similarly for a 1 cm meteoroid τ_{PR} is ten times longer, τ_Z twenty times longer.

For each of the 28 stream orbits the lifetimes are listed in Table 4; τ_S is not available for several of these, as noted in section 5.3. It should be remembered that these are generally in order of increasing semi-major axis. For an easier comparison of the data, the lifetimes are plotted in Figure 6 against the stream number (1 to 28 from Table 4): τ_D is excluded since it is of little importance individually, but is incorporated by τ_A.

6.2 DEDUCTIONS

The deductions which can be made from Figure 6 are fairly obvious, and few comments are needed. Briefly, these are as follows:

(a) For those orbits which do not cross Jupiter (streams 1-10), the evolution of 1 mm meteors is dominated by collisions with zodiacal cloud particles which occur on a time-scale $\sim 10^5$ years. For 100 μm meteors the roles are reversed and $\tau_{PR} < \tau_Z$; thus the meteoroid will spiral in a considerable distance towards the sun before being struck down. For a 1 cm meteor τ_{PR} and τ_Z will be comparable, and on average some shrinkage of its orbit will occur before an eventual fate in a collision.

(b) Sporadic meteor production by terrestrial planet encounters is insignificant, occuring on a time-scale τ_A, $\tau_S > 100\ \tau_Z$. Such stream disruption in the inner solar system as produces sporadic meteors therefore must be a result of inter-particle collisions.

TABLE 4: Lifetimes for various stream orbits

Number	Stream Name	τ_L	τ_D	τ_S	τ_A	τ_Z	τ_{PR}
		(All lifetimes in years)					
1	Geminids	2.4E8	2.8E3	7.3E8	1.3E8	6.4E4	1.0E5
2	Camelopardalids	6.5E7	1.4E3	2.0E6	1.1E5	7.6E4	2.3E6
3	Daytime Zeta Perseids	1.4E6	1.1E1	1.7E6	1.5E5	8.1E4	4.3E5
4	Daytime Arietids	2.8E8	3.2E3	8.7E8	2.1E8	8.1E4	6.2E4
5	Northern Iota Aquarids	7.7E7	9.3E2	2.1E8	2.3E7	9.3E4	3.3E5
6	Southern Iota Aquarids	1.4E8	1.9E3	4.3E8	9.1E7	1.5E5	2.4E5
7	Daytime Beta Taurids	1.3E8	1.6E3	3.8E8	3.7E7	1.3E5	4.8E5
8	Alpha Capricornids	2.2E8	2.5E3	1.2E8	1.2E7	1.6E5	1.2E6
9	Northern Taurids	9.3E7	9.4E2	2.5E8	2.1E7	1.7E5	5.9E5
10	Southern Taurids	9.8E7	1.2E3	3.2E8	1.9E7	1.1E5	5.5E5
11	Northern Delta Aquarids	1.7E6	1.6E2	-	1.1E3	1.7E5	5.8E4
12	Southern Delta Aquarids	2.1E6	2.1E2	1.7E4	1.9E3	1.9E5	4.9E4
13	Virginids	2.3E5	2.8E1	-	1.0E2	1.7E5	3.6E5
14	Theta Ophiuchids	2.2E5	3.6E1	1.1E3	8.8E1	2.0E5	9.2E5
15	December Phoenicids A	2.0E6	2.7E2	-	3.0E2	2.0E5	2.9E6
16	December Phoenicids B	3.6E6	3.8E2	-	3.8E2	2.0E5	3.0E6
17	Quadrantids	3.1E6	3.0E2	-	3.9E3	2.2E5	3.1E6
18	Kappa Cygnids	1.8E6	3.1E2	-	9.9E2	2.2E5	3.1E6
19	October Draconids	2.2E6	4.6E2	2.1E4	1.7E3	2.6E5	3.3E6
20	Andromedids	1.1E6	2.5E2	9.6E3	7.6E2	2.7E5	2.6E6
21	Pegasids	7.3E5	2.0E2	8.8E3	7.0E2	3.0E5	3.3E6
22	Ursids	7.9E6	1.1E3	1.8E5	2.8E4	5.4E5	3.2E6
23	Leonids	8.1E6	8.7E2	2.2E5	1.7E5	1.6E6	5.4E6
24	Eta Aquarids	6.9E6	1.2E3	2.4E5	2.0E5	1.9E6	2.4E6
25	Orionids	6.7E6	1.4E3	2.7E5	2.5E5	2.3E6	2.7E6
26	Perseids	1.5E7	1.1E4	1.3E6	2.0E6	5.9E6	8.3E6
27	April Lyrids	1.1E7	1.5E4	1.4E6	1.5E6	5.9E6	7.2E6
28	Monocerotids	4.5E6	1.3E4	8.5E5	2.5E6	1.1E7	4.6E5

FIGURE 6

Lifetimes for various stream orbits

The following five lifetimes are plotted against stream number (1 to 28, taken from the sequence in Tables 3 and 4):

τ_L (denoted L): the time-scale for loss due to planetary collision or ejection from the solar system, equation 81.

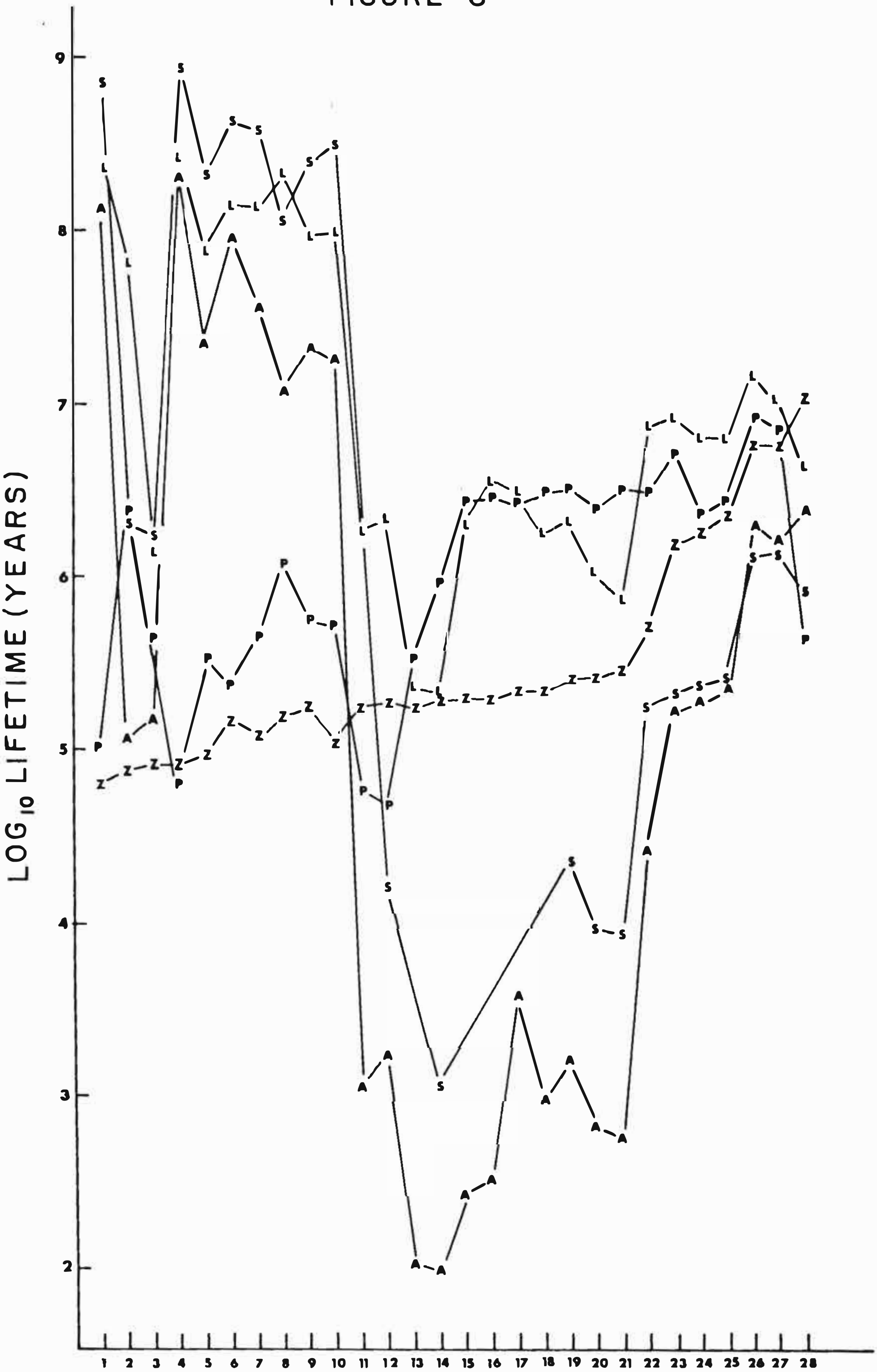
τ_S (denoted S): the time-scale for sporadic production using equation 84.

τ_A (denoted A): The time-scale for sporadic production using equation 83.

τ_Z (denoted Z): the lifetime against inter-particle collisions for a 1 mm meteoroid from equation 91.

τ_{PR} (denoted P): the Poynting-Robertson lifetime from equation 87 for a radius of 1mm.

FIGURE 6


LOG10 LIFETIME (YEARS)
STREAM NUMBER
9
8
7
6
5
4
3
2
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28

(c) Loss of meteoroids due to planetary collisions occurs much more slowly than loss due to interparticle collisions, since $\tau_L \sim 10^3\ \tau_Z$; τ_L is purely a result of collisions since no ejection is possible by the terrestrial planets (Table 3).

(d) As soon as the orbit under consideration is Jupiter-crossing (streams 11-28), sporadic meteor production (τ_A or τ_S) occurs much faster than any of the loss mechanisms.

(e) For the larger orbits the most important loss mechanism is still catastrophic collisions. (Since the majority of these collisions occur within 1 A.U., this comment would not necessarily apply to a stream which had perihelion outside of the Earth's orbit: naturally such a stream is not observable by ground-based meteor techniques). An exception could be the smallest meteoroids (radius 100 μm) in the largest orbits (a > 10 A.U.), for which $\tau_{PR} < \tau_Z$.

(f) For those orbits which cross the giant planets, the loss rate for particles larger than 1 mm due to collisions and ejections in planetary encounters is comparable to that due to Poynting-Robertson drag. The Poynting-Robertson drag would be dominant for smaller meteoroids.

(g) Stream orbits 11 to 21 produce sporadic meteors on a time-scale of the order of 10^3 years. These orbits have semi-major axes ranging from 2.6 to 3.9 A.U. All cross Jupiter, but with a low relative velocity so that the angular deflection of the meteoroid is large: this, along with the short periods of such orbits, causes the stream dispersal into sporadic orbits on such a short time-scale.

(h) Some idea of the shower-to-sporadic ratio can be gained from Figure 6. For streams 13 to 21, $\tau_A \sim 10^3$ years and $\tau_L \sim 10^6$ years. The size-dependant loss times are:

100 μm	τ_Z	$\sim 10^6$ years	
	τ_{PR}	$\sim 10^5$ years	$(= 100\ \tau_A)$
1 mm	τ_Z	$\sim 10^5$ years	$(= 100\ \tau_A)$
	τ_{PR}	$\sim 10^6$ years	
1 cm	τ_Z	$\sim 10^6$ years	$(= 1000\ \tau_A)$
	τ_{PR}	$\sim 10^7$ years	

Therefore it appears that sporadic production from such streams occurs about a hundred times faster than losses for the smaller meteoroids, but a thousand times faster for 1 cm bodies. This is not reconcilable with the observational evidence which shows about 50% of ($\sim$ 1 cm) photographic meteors to be stream members, but 25% or less of ($\sim$ 1 mm) radar meteors (Sekanina, 1977), indicating that some other factor must be removing larger meteoroids.

6.3 SUMMARY

It has been shown that meteor streams with orbits similar to short-period comets ($a < 5$ A.U., $e > 0.7$, low inclination: streams 13 to 21) can produce sporadic meteors as a result of close encounters with Jupiter at a rate two orders of magnitude faster than they are removed from the interplanetary complex. This is consistent with the fact that the majority of streams have orbits similar to short-period comets, but the majority of sporadic meteors have orbits of semi-random eccentricity and perihelion distance, and random inclination well away from the ecliptic (Lebedinets, 1968; Hughes, 1978). Such a distribution would be expected as a result of planetary close-encounters, whereas inter-particle collisions would tend to decrease inclinations and eccentricities. Although after a deflection by Jupiter a sporadic meteor remains in an orbit crossing that planet, other influences (Poynting-Robertson, erosional impacts) would cause it to lose orbital energy until its aphelion distance was within 5 A.U. It would then remain a member of the sporadic complex on a relatively constant orbit until lost in a catastrophic collision. Catastrophic collisions with zodiacal cloud particles occur on a time-scale of $\gtrsim 10^5$ years and so do not make a large contribution to sporadic production; erosive collisions with much smaller particles (radius ~ 10 μm) resulting in small changes in the orbit of the meteoroid would occur on a shorter time-base and could be of some significance.

CHAPTER 7

APPLICATION TO OTHER SOLAR SYSTEM SCENARIOS

7.1 INTRODUCTION

Since the theory of chapters 2 and 3 is applicable to any orbit where the particle mass and radius is much smaller than those of the scattering planet, it is of interest to apply this theory to various solar system objects. Such objects include known comets and asteroids, and also Pluto which crosses the orbit of Neptune (the mass of Neptune is about 5000 times that of Pluto).

In this chapter I apply the close encounter theory to a number of asteroids, and to Pluto, but do not consider any comets (i.e. high eccentricity test orbits) because this has effectively been accomplished by the preceding chapters: half of the 28 chosen meteor streams have well-known parent comets moving in similar orbits. For example (Drummond, 1981):

Encke (1971 II):	Beta Taurids, N. Taurids, S. Taurids
Blanpain (1819 IV):	December Phoenicids
Giacobini-Zinner (1946 V):	October Draconids
Biela (1852 III):	Andromedids (Bielids)
Tempel-Tuttle (1965 IV):	Leonids
Halley (1835 III):	Eta Aquarids, Orionids (Halleyids)

In addition, a small asteroid has recently been discovered in the orbit of the Geminids (Hughes, 1983; Fox et al, 1984). By implication this is the expended core of the Geminid parent comet, and also strongly indicates comets to be the progenitors of the Apollo-Amor-Aten asteroids. Recent reviews of the dynamics of asteroids and their relationship to comets have been by Kresak (1979; 1980; 1984).

In the next two sections I consider those peculiar asteroids in orbits which cross the giant planets. In section 7.4 the case of Pluto is investigated. Asteroid collision rates with each of the terrestrial planets are evaluated in Appendix 2.

7.2 HIDALGO AND CHIRON

7.2.1 Previous Work

Two large bodies are known to cross the orbits of the giant planets: (944) Hidalgo and (2060) Chiron. Although nominally classed as asteroids, hence their numbering as minor planets, in many ways the two are cometary in nature (Kresak, 1979; Marsden, 1980).

By assuming a low, asteroidal, albedo for Hidalgo a diameter ranging from 39 to 60 kilometres results (Kresak, 1979; Degewij and van Houten, 1979). Chiron is one of the eight largest known asteroidal bodies: an albedo of 0.5 would give it a diameter of 100 kilometres, but a more likely albedo of 0.05 renders a diameter of 320 kilometres

(Kowal, 1979). Hartmann et al (1981) find a diameter in the range 310 to 400 kilometers. It is possible that Chiron is similar in nature to Phoebe, Saturn's outer moon (Kowal et al, 1979; Hartman et al,1981).

Many other asteroids are known to have aphelia at or beyond the orbit of Jupiter, such as the Trojan and Hilda families. However, stable resonances and librations prevent their close approach to the planet (Kresak, 1979), so that a collision or severe perturbation is not possible. Since Hidalgo crosses both Jupiter and Saturn, and Chiron crosses both Saturn and Uranus, no such stability is possible: their orbits have been described as being 'chaotic', with an eventual fate of collision with a planet or ejection from the solar system (Marsden, 1970; Kowal, 1979; Kowal et al, 1979; Oikawa and Everhart, 1979; Everhart, 1979; Scholl, 1979).

Hidalgo and Chiron are therefore only temporary occupants of their present orbits, with a lifetime measured in units of only 10^4 or 10^5 years (Marsden, 1980). Chiron's origin might lie in the asteroid belt, having been thrown outwards by approaches to Jupiter, or it may be gradually moving inwards from the outer reaches of the solar system (Smith, 1978, 1980; Kowal, 1979). A similar career might be postulated for Hidalgo.

There are a variety of ways in which the possible orbital evolution of these bodies can be investigated. Their osculating elements are well known (Hidalgo was discovered in 1920, and although Chiron was not identified until 1977, pre-discovery positions back to 1895 are known). The residuals from their expected positions have been used to determine the masses of Saturn and Uranus (Landgraf, 1983). By performing numerical integrations of the actual orbits over a period of about 15,000 years, Oikawa and Everhart (1979; see also Everhart, 1979) found numerous close approaches of Chiron to both Saturn and Uranus. They found that there was a 1 in 8 chance that Saturn would eject Chiron from the solar system within $\sim 3 \times 10^6$ years; otherwise the orbital evolution would be inwards, most probably to interact with Jupiter. Similar (but not identical) results were found in this way by Scholl (1979): exact agreement would not be expected, this technique being only of statistical significance.

Arnold (1965) used a Monte Carlo simulation based upon the encounter probability equations of Öpik (1951) to investigate the possible orbital evolution of Hidalgo. By following the paths of 500 objects with initial elements similar to those of Hidalgo, Arnold found a 73% probability of ejection by Jupiter, 5% for ejection by Saturn, and a single capture by Jupiter. The remaining bodies were thrown into orbits of small perihelion distance. The median lifetime of these Hidalgo-like objects before severe perturbation was $\sim 3 \times 10^5$ years, although Arnold noted a large spread. Arnold's method (1964; 1965; see Dohnanyi, 1978 for an explicit description) consisted of a series of steps in which a random choice was made between possible planetary encounters, and the precise geometry of the encounter. The changed orbit thus produced was then used as a new initial orbit, and the object followed until its eventual fate. This method was developed in order to investigate the origin of large meteorites, and is superior in some ways to the technique used here since it allows for the gradual evolution of the particle orbit. However, to get a statistically valid result, a large number of test particles must be run in Arnold's method, entailing considerable computation time.

The case of Hidalgo was investigated by Öpik (1963) using a probabilistic method similar to that used here. He found a lifetime of 2.3×10^6 years for physical loss of the asteroid either by planetary collision or by ejection from the solar system. It has been shown (Weidenschilling, 1975a) that Öpik's expression for the ejection probability was incorrect; however a lifetime of this order may be expected.

7.2.2 Present Results

Using the orbital elements of Hidalgo and Chiron from the TRIAD file (Bender, 1979) the results obtained, by means of the techniques detailed in Chapters 2 and 3, are shown in Table 5.

The time between planetary encounters by Hidalgo is $\tau_D \simeq 10^3$ years, each encounter producing a deflection of a few degrees; it therefore has a rapidly evolving orbit. Its lifetime against collision or ejection is $\tau_L \simeq 4 \times 10^6$ years, or about one-thousandth of the age of the solar

TABLE 5: CLOSE ENCOUNTERS BY HIDALGO AND CHIRON TO THE GIANT PLANETS

	(944) HIDALGO	(2060) CHIRON
Semi-major axis =	5.86 A.U.	13.70 A.U.
Eccentricity =	0.6565	0.3786
Inclination =	42.40 degrees	6.92 degrees
Period =	14.19 years	50.71 years

	JUPITER	SATURN	SATURN	URANUS
Probability (per year) of Collision:	9.22E-09	3.93E-09	1.70E-08	5.76E-10
Deflection:	6.00E-04	4.62E-04	3.72E-04	8.38E-05
Ejection:	2.26E-07	1.36E-08	-	-
Velocity(km/s) Minimum:	11.7	6.3	2.3	1.6
Maximum:	13.5	7.9	3.8	2.1
Fractional Energy Change (per year):	-1.74E-06	-6.52E-07	+4.43E-06	0.00E+00
Deflection, Mean (deg.):	2.056	1.579	8.469	3.257
RMS (deg.):	3.216	2.536	11.557	4.929

system: thus it is untenable that Hidalgo originated in this region. τ_L is dominated by the probability of ejection by Jupiter, this being about 17 times more likely than Saturn ejection. This figure is in line with Arnold's (1965) value of 15:1. Based upon its present orbit, the rate of orbital energy change due to close encounters shows that Hidalgo should gradually move outwards on a time-scale of 10^5 - 10^6 years.

Chiron cannot be ejected in the present epoch since its relative velocity in an encounter with either Saturn or Uranus is less than $(2^{\frac{1}{2}} - 1)$ times the circular orbit velocity. The loss lifetime is hence due only to collisions, and is substantial ($\tau_L \simeq 6 \times 10^7$ years). The time between encounters is $\tau_D \simeq 2 \times 10^3$ years, confirming the results of Oikawa and Everhart (1979) and Scholl (1979). The deflections produced are so high that Chiron would rapidly attain an orbit for which the ejection probability was no longer zero; even if this were not so it is highly unlikely that an orbit such as Chiron's could have avoided a collision with Saturn over the age of the solar system. The indication from the orbital energy change is that Chiron should gradually decrease its semi-major axis, as found by Oikawa and Everhart (1979) and Scholl (1979).

The overall picture is therefore of Hidalgo and Chiron being transient phenomena in the outer solar system. The pair may be indicative of the larger bodies in a population of asteroidal objects which exist in that region, or they may be examples of huge extinct comets although, as discussed by Kresak (1979), for the case of Chiron's orbit it would never have been sufficiently close to the Sun to have been active. Their relationship to the satellites of the outer planets is also of interest. An alternative scenario (Smith, 1978; 1980) is that these are perhaps asteroidal bodies which have been thrown outwards from the asteroid belt.

7.3 OTHER ASTEROIDS WHICH CROSS THE GIANT PLANETS

In addition to Hidalgo and Chiron, four smaller asteroids are known whose orbits are thought to cross one of the giant planets without being protected from close approach by a commensurability. These are 1939 TN, 1982 YA, 1983 SA, and 5025 P-L. The first three were discovered in the years indicated by their names (although 1939 TN has a recently updated orbit), whilst 5025 P-L was found recently on plates taken in 1960. 1982 YA and 1983 SA have well-known orbits, but knowledge of the other two is quite vague and they are not recoverable. The present values of their orbital parameters are as in Table 6. If these elements are correct, then 5025 P-L crosses all of the planets from Mercury to Jupiter, although the discoverers caution that only three observations spaced over seven days were used in determining its orbit (van Houten et al, 1984). Similarly for 1939 TN only four observations over a month were used.

TABLE 6: ORBITAL PARAMETERS OF FOUR JUPITER-CROSSING ASTEROIDS

	a=(A.U.)	e=	i=(degrees)	reference
1939 TN	4.52	0.2481	2.12	Marsden and Bardswell (1982)
1982 YA	3.71	0.6973	34.57	MPC* 8534
1983 SA	4.23	0.7147	30.78	MPC* 8678
5025 P-L	4.20	0.8954	6.20	van Houten et al (1984)

*Minor Planet Circulars, issued by the Minor Planet Center, Cambridge, Massachusetts.

As is to be expected, Jupiter dominates the orbital evolution of these objects. Although all except 1939 TN can intercept Mars, and 5025 P-L the inner three planets also, the probability of a collision with any of the terrestrial planets is at least two orders of magnitude smaller than for a Jupiter impact. Venus and the Earth can also eject 5025 P-L, but the chance of this is even smaller than the collision probability. In view of the above, only the Jupiter-encounter probabilities are of interest, and these are given in Table 7.

The deflection time-scale (τ_D, Table 7) is less than a thousand years for all four of these asteroids, with an expected deflection of at least a few degrees. The loss time-scale for ejection or planetary collision (τ_L) is at least a thousand times longer; their lifetime against destruction in a collision with belt asteroids is of the order of 10^7 years or more (Wetherill, 1976). Many encounters are therefore expected over their lifetime, with a later orbit bearing little similarity to the original elements. Hidalgo and Chiron could have originated in this way, and are probably indicative of a large population of asteroids in the outer solar system whose orbits have diffused outwards. These two bodies have been detected because of their large size compared to 1939 TN, 1982 YA, 1983 SA and 5025 P-L which are only ~ 1 km in diameter. Smith (1978) suggested that there might be several hundred small asteroids with orbits between the giant planets: the comparative time-scales derived here bear this suggestion out, given a sufficient supply of exhausted Jupiter-crossing comets.

TABLE 7: ENCOUNTER PROBABILITIES WITH JUPITER FOR FOUR ASTEROIDS

Asteroid	P_C=	P_D= (all per year)	P_E=	χ_{RMS}= (degrees)	τ_D=	τ_L= (years)
1939 TN	5.5E-6	1.5E-2	-*	43	67	1.8E5
1982 YA	3.9E-8	1.6E-3	3.4E-7	4.8	625	2.6E6
1983 SA	2.6E-8	1.3E-3	3.1E-7	4.2	770	3.0E6
5025 P-L	8.4E-8	5.6E-3	1.0E-6	3.1	180	9.2E5

*Due to its low eccentricity orbit, 1939 TN approaches Jupiter with a maximum velocity of only 3.6 km s^{-1}, and cannot therefore be ejected. This also leads to its large r.m.s. deflection.

7.4 NEPTUNE AND PLUTO

7.4.1 Introduction

It is not possible for Neptune and Pluto to collide. An early consideration (Lyttleton, 1936) of the results of an encounter between the newly-discovered planet and the Neptunian system was later shown to be untenable since the two bodies never come closer than 18 astronomical units (Cohen and Hubbard, 1965); this is because the 3:2 commensurability in their orbital periods results in Pluto always being close to aphelion when in conjunction with Neptune. Additionally, conjunction is close to 90° in longitude from the mutual orbit node (Williams and Benson, 1971). Hence although Pluto is the only planet to cross the orbit of another, it is not threatened by obliteration, orbital disruption or capture by Neptune, at least in the present epoch. In fact, its orbital is remarkably stable with respect to gravitational perturbation (Peale, 1976; Greenberg, 1977; Nacozy, 1980).

Despite the fact that the planets now avoid each other, this scenario cannot necessarily be extrapolated back to the early solar system. The pecularity of Pluto's orbit and the chaotic nature of the Neptunian satellite system has led to various suggestions for the origin of Pluto, mostly involving a close encounter with Triton which throws Pluto out into its lonely heliocentric path.

The discovery of a moon of Pluto - or more correctly that Pluto is a binary planet - has led to the imposition of added constraints upon the evolution of the Pluto-Charon system. The two must have been joined together when in orbit about Neptune if this were their genesis, with separation later by binary fission on account of a high spin rate (Lin, 1981; Mignard, 1981). This would strengthen the supposition of an origin elsewhere. There has also been some debate as to whether another object, now unseen but several times the mass of the Earth, would be necessary to eject Pluto and throw the Neptunian moons into confusion (Farinella et al, 1978, 1980; Harrington and Van Flandern, 1979). Mass-loss by a proto-Neptune causing Pluto to be thrown out into a large heliocentric orbit has been discounted (Horedt, 1974).

Laying aside the possibility that Pluto was originally a Neptunian satellite (McKinnon, 1982), there are three broad categories of explanation for its enigmatic presence:

(i) It was directed into its stable orbit during the early history of the solar system, when the presence of an appreciable nebula allowed for the dissipation of orbital energy and angular momentum (Dormand and Woolfson, 1980); or

(ii) The 3:2 resonance may have favoured the aggregation of a planet with this semi-major axis; or

(iii) More than one object existed at first, with Pluto being the sole survivor.

Here I will evaluate the probability of a chance encounter between Neptune and an object in a Pluto-like orbit in order to find whether a multitude of such objects could have existed originally, as in (iii) above, with all except for Pluto having been lost due to collisions with, or orbital disruptions by, Neptune. In order to avoid any confusion, I will call this Pluto-like object 'Zagreus', for the son of Zeus and Persephone, who as a child was torn asunder by the Titans.

7.4.2 Application to Pluto

Because of its resonance with Neptune, Pluto cannot take any position in its orbit but is confined. I will consider Zagreus to have a random argument of perihelion and longitude of node, with orbital elements (a,e,i) identical to the present mean values of Pluto (Seidelmann et al, 1980):

$$a = 39.72 \text{ A.U.}$$

$$e = 0.2524$$

$$i = 17\overset{\circ}{.}14$$

The mass and radius are about 1.6×10^{22} kg and 1600 km respectively (Lupo and Lewis, 1980; Morrison et al, 1982). Since this makes Zagreus much smaller than Neptune, it can be considered as a point mass.

Öpik (1951) found a collisional lifetime of 1.03×10^{10} years for Pluto, or a collision probability of a little under 1×10^{-10} per year. In one way Öpik's method is a good approximation for this circumstance in that the orbit of Neptune has a very low eccentricity, and hence is almost circular. However, Zagreus is a very shallow crosser of Neptune's orbit so that a small change in the eccentricity and semi-major axis will make a large difference to the collision probability.

Despite the latter comment, the collision probability obtained by the method of Chapter 2 is close to that found by Öpik (1951):

$$P_C \simeq 1.14 \times 10^{-10} \text{ per year}$$

so that the collision lifetime, in units of the age of the solar system (4.6 billion years) is around 1.9. Thus consideration of mere collisions is not able to tell much about any primordial population with the orbit of Pluto: if the lifetime were 0.1 then almost all such objects would have been collisionally lost, if it were 10 then almost none would have been lost.

Attention is now directed to the result of an orbital disruption in a close encounter. Since the relative encounter velocities range from 1.7 to 2.0 km s^{-1} it is not possible for Neptune to eject Zagreus on an hyperbolic orbit: the orbital velocity of Neptune is 5.4 km s^{-1} so that a relative velocity of at least 2.24 km s^{-1} would be necessary. Thus $P_E = 0$ here.

The probability of passage within the sphere of influence is

$$P_D \simeq 4.20 \times 10^{-5} \text{ per year}$$

or

$$\tau_D \simeq 2.4 \times 10^4 \text{ years} .$$

Therefore over the age of the solar system it would not be possible for Zagreus to avoid frequent close approaches to Neptune, unless in a protected position such as Pluto.

Deflections of the order of $\chi_{RMS} = 3\overset{\circ}{.}7$ result in a fractional rate of energy change (equation 78) of -2.9×10^{-8} per year. About half of the orbital energy must be lost if Zagreus is to become a Uranus-crosser, and thence be passed on to control by Saturn and Jupiter. Migration inwards due to close encounters by Zagreus therefore occurs on a time-scale of $\sim 10^8$ years, and is a possible alternative origin for Chiron and Hidalgo.

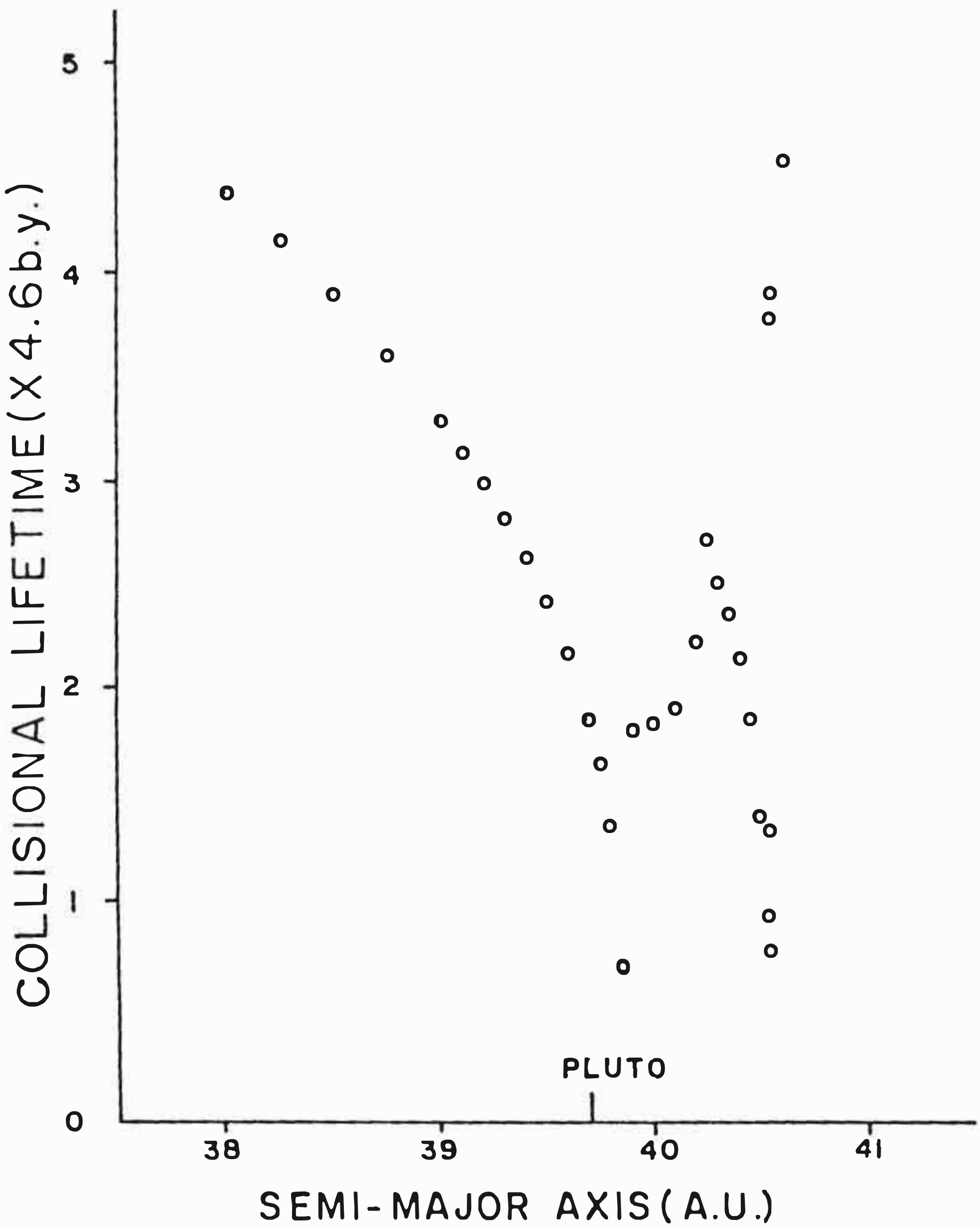
In view of the above it is apparent that Pluto only survives in its present orbit because it is protected from close encounters with Neptune. Any objects with similar but non-commensurable orbits would have been disrupted over astronomical time.

Two further points of interest can be mentioned here. Firstly, by putting Zagreus into a similar but retrograde orbit ($i = 162\overset{\circ}{.}86$) the collision probability is reduced to $P_C \sim 2.3 \times 10^{-11}$ per year since the encounter velocity goes up by a factor of six, to 11.3 km s^{-1}, and the gravitational radius is correspondingly reduced. However, ejection is now possible with a probability $P_E \simeq 1.9 \times 10^{-10}$ per year. Therefore physical loss (collision or ejection) is about twice as probable for this retrograde orbit.

Secondly, the collision (or encounter) probability changes markedly as either the semi-major axis or eccentricity is altered. This is to be expected since the region of orbital overlap changes significantly. For convenience, in Figure 7 I have plotted the collisional lifetime in units of the age of the solar system against the semi-major axis of Zagreus, with the eccentricity and inclination held at the nominal values; a similar curve results from varying the eccentricity instead. Although the exact shape of the curve depends upon the size of the volume elements (equation 7), two distinct minima are always seen. These are at 39.85 and 40.54 A.U., which with $e = 0.2524$ give Zagreus a perihelion distance of 29.80 and 30.31 A.U. respectively. The latter figures correspond to the perihelion and aphelion distances of Neptune, and are therefore easily explained: the spatial density for both planets would be enhanced at these distances, so that the collision probability is higher. The present orbit of Pluto, as indicated in Figure 7, puts it close to one of the lifetime minima (probability maxima). Secular orbital variations (Cohen and Hubbard, 1965; Williams and Benson, 1971) can lead to it crossing both of the lifetime minima.

To summarize, Pluto only exists in its present orbit because it is commensurable with Neptune and in a position such that the larger planet is avoided. It is possible that a number of bodies in similar orbits once existed, with Pluto being the sole survivor due to this condition.

FIGURE 7: VARIATION OF THE COLLISIONAL LIFETIME OF ZAGREUS WITH SEMI-MAJOR AXIS



CHAPTER 8

CONCLUSIONS

In this thesis I have investigated the factors which are important in determining the orbital characteristics of meteoroids. In recent years much work has been done on the effects of the solar radiation field upon interplanetary dust (chapter 4), but comparatively little upon encounters with other bodies. Although it has been recognized since 1975 that interparticle collisions are the eventual fate of most meteoroids, no satisfactory method has been published which allows the characteristic lifetime against such events to be calculated as a function of the orbital elements (a,e,i) of the meteoroid. In chapter 4, using the collision probability theory of Kessler (1981) developed in chapter 2, I have shown how this lifetime can be found and have calculated values for various sample orbits. The answers are similar to those from previous, rudimentary, estimates. This new method is more realistic than previous attempts, and is easily applied.

The major thrust of this thesis, however, has been an investigation of meteoroid encounters with larger bodies: the planets. The majority of these encounters do not result in a collision but in a gravitational disruption of the meteoroid orbit. In order to find the gross effects of such orbital changes, various parameters (r.m.s. deflection angle, probability of ejection from heliocentric orbit etc.) were calculated using the method of Weidenschilling (1975a) as developed in chapter 3. A computer program written to accomplish this, and encompassing the techniques of chapters 2 and 3, is included here as Appendix 1. Use of this program upon 28 sample orbits, corresponding to most of the well-known meteor streams, allowed the effect of planetary close encounters to be gauged. Comparing the loss lifetime due to planetary collisions or ejections (τ_L) and the time-scale for sporadic meteor production (τ_A) calculated with this program against the loss lifetime due to the Poynting-Robertson effect (τ_{PR}) and collisions with zodiacal cloud particles (τ_Z), I have shown that (chapter 6):

(a) τ_Z is indeed the limiting lifetime for all but a few meteoroids;

(b) τ_{PR} and τ_L are comparable for a large meteoroid (radius $\gtrsim$ 1 mm) in a Jupiter-crossing orbit; this is a new and unexpected result;

(c) Sporadic meteoroid production by gravitational diffusion of streams in orbits similar to short-period comets occurs much faster than any of the loss mechanisms ($\tau_A << \tau_L, \tau_Z, \tau_{PR}$) and is therefore the major source of sporadic meteors; this is an important deduction, answering a question which has been posed on many occasions.

In addition to applying these techniques to meteoroid orbits, for which purpose this investigation was started, I have also shown in chapter 7 the results obtained for some of the larger bodies which cross the orbits of the major planets. The present orbits of six asteroidal bodies have been found to be extremely short-lived. The planet Pluto I have shown to be in an orbit which would also be very short-lived if it were not that Pluto is in a stable resonance with Neptune.

Finally in Appendix 2 I have applied this collision probability theory to the known population of Apollo-Amor-Aten asteroids and calculated the influx of these bodies (diameter $\gtrsim$ 1 km) to each of the terrestrial planets, from which the production rate of large impact craters on each can be found.

One last point can be made here. For many years it was erroneously believed that the major source of meteoroids was the capture of interstellar particles by planetary close encounters (e.g. Lovell, 1954, for a discussion). As a result of exhaustive Monte Carlo simulations it is well-known that periodic comets can be captured in this way, although multiple distant perturbations are mostly responsible (Everhart, 1973). By applying the principle of reversibility to the particle ejections studied here, the chance of capture is also found i.e. an upper limit for the probability of capture into an elliptical heliocentric orbit from an unbound trajectory is just given by the probability per perihelion passage of ejection from the solar system. The results obtained herein therefore confirm that few particles of any kind are captured by the solar system as a consequence of close planetary encounters.

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APPENDIX 1

PLANETARY ENCOUNTER PROGRAM

This appendix consists of a listing of the program written to perform the encounter probability calculation of chapter 2 and the encounter result analysis of chapter 3; the program is named CDE (Collision, Deflection, Ejection). The language used was PASCAL, in conjunction with the University of Sheffield PASCAL compiler version 3.0 on the University of Canterbury PRIME 750 computer.

This particular version of the program is the most time-consuming of all versions since it calculates the ejection probability (equation 64) for each volume element in the summation represented by equation (7). Although each volume element is associated with four distinct relative velocities (equation 34) on account of the four possible encounter angles (equation 37), these do not often vary appreciably and therefore the ejection probability can usually be adequately estimated from a few evaluations of (64) using typical encounter velocities and trajectories. Alternate versions of this program with minor changes have been prepared, with large savings in computer time because of the above simplification. The present program requires 11 minutes of C.P.U. time on the PRIME computer for the case of the Monocerotids, which is the only one of the 28 stream orbits used in Chapter 5 which can be ejected by all 8 planets. The majority of these streams require less than 1 minute of

C.P.U. time, reducible by a factor of more than 10 if (64) is not evaluated for every volume element.

```
PROGRAM ODE(INPUT,OUTPUT);

                    {August 10th 1984}
{This program:
 a) Finds the encounter probability between an arbitrary object in
    heliocentric orbit and each of the planets;
 b) From this determines the collision probability, the ejection
    (from the solar system) probability, and the mean and RMS
    deflection in an encounter.
 The encounter probability is found using the method of Kessler
 (Icarus, 48, 39-48, 1981) and the results of the encounter
  using Weidenschilling (Astron.J., 80, 145-153, 1975).          }

CONST    PI = 3.14159265;
     DEGRAD = 0.017453293;
      ONEAU = 1.4960E11 ; {One astronomical unit in metres}
        RT2 = 1.4142136 ;
VAR     CFILE                      : TEXT;
        CNAME                      : ARRAY[1..30] OF CHAR;
        NAMEBIT                    : CHAR;
        R,RD,B,BD,RBAR,VOLUME      : REAL;
        D1,D2,D3,D4,DUMMY          : REAL; {Dummy variables}
        A1,A2,E1,E2,I1,I2,S1,S2    : REAL;
        DELTAR,Q1,Q2,QD1,QD2       : REAL;
        ISMALL,IBIG,QSMALL,QBIG    : REAL;
        I11,I22,JUMPR,JUMPB        : REAL;
        RATGMONE,TWORBAR,RAD1SQ    : REAL;
        RAD1,MASS1,VESQ,IMETEOR    : REAL;
        VCIRC,VLEAST,VMOST,VRC     : REAL;
        USIGC,USIGD,USIGE          : REAL;
        SUMC,SUMD,SUME             : REAL;
        TCP,TDP,TEP,VMIN,VMAX      : REAL;
        ALF1,ALF2,V1,V2,GMSUN      : REAL;
        SG1,SG2,CG1,CG2            : REAL;
        CHIRMS,CHIBAR,VREL         : REAL;
        I,J,K,JRANGE,KRANGE,PNO    : INTEGER;
        PSMA,PECC,PINC,PRAD,
        PMASS,MINV,MAXV,POOL,
        PDEF,PEJE,DKDT,DBAR,DRMS : ARRAY[1..8] OF REAL;

{*****************************************************************}

FUNCTION ASIN(XYZ : REAL) : REAL;
{Returns the inverse sine of parameter XYZ}
BEGIN
IF ABS(XYZ) > 0.999999 THEN ASIN:=XYZ*PI/2.0
     ELSE ASIN:=ARCTAN(XYZ/SQRT(1.0-(XYZ*XYZ)));
END;   {of function ASIN}

{*****************************************************************}

FUNCTION ACOS(ABC : REAL) : REAL;
{Returns the inverse cosine of parameter ABC}
BEGIN
IF ABS(ABC) > 0.999999 THEN BEGIN
                          IF ABC<0.0 THEN ACOS:=PI
                                     ELSE ACOS:=0.0;
                            END
    ELSE ACOS:=(PI/2.0) - ARCTAN(ABC/SQRT(1.0-(ABC*ABC)));
END; {of function ACOS}

{*****************************************************************}
```

```
PROCEDURE LIMIT(VAR X789 : REAL);
{This procedure limits X789 to be in the range -1 to +1}
BEGIN
IF (X789 > 1.0) THEN X789:=1.0;
IF (X789 < -1.0) THEN X789:=-1.0;
END; {of procedure LIMIT}

{****************************************************************}

PROCEDURE SBSI(VAR PA : REAL; PB,PC : REAL);
{This procedure finds  Sin(Latitude) / Sin(Inclination)
 and assigns PA the value of this ratio.
 PC is the Sin(Lat.), and PB is the Sin(Incl.) .
 Whenever the Inclination is very small, PA is set to
 plus or minus one, depending upon whether the latitude
 is North or South. The only exception to this is when
 the latitude is also small (zero) : then PA=0    }
BEGIN
  IF PB < 1.0E-10 THEN BEGIN
     IF PC < 0.0 THEN PA:=-1.0 ELSE PA:=1.0;
     IF ABS(PC) < 1.0E-10 THEN PA:=0.0;
                        END
                       ELSE PA:=PC / PB ;
  LIMIT(PA);
END;     {of procedure SBSI}

{****************************************************************}

PROCEDURE CHIS(VAR CHIG,CHID,D : REAL; MP,RP,U,RV : REAL);

{This procedure calculates the values of the maximum non-impact
 deflection CHIG (i.e. grazing) and the minimum deflection CHID at
 the edge of the sphere of influence, radius D .
 Input parameters are:
       MP = mass of planet in E20 kg                        ;
       RP = radius of planet in km                          ;
       U  = relative velocity of particle and
                planet in m/s                               ;
       RV = radius vector, Sun to planet, in A.U.           }

VAR   X1,GMPOU2    : REAL;
      A1,A2,A3,A4  : REAL;  {Dummy variables}
      B1,B2,B3,B4  : REAL;  {Dummy variables}
BEGIN
GMPOU2:=6.672041E9*MP/SQR(U); {G * mass planet/square of
                          relative velocity, in SI units}
X1:=GMPOU2/(RP*1000.0); {Again in SI units}
CHIG:=2.0*ARCTAN(X1/SQRT(1.0+(2.0*X1)));
X1:=MP/(3.32958*5.9742E9); {The planet to Sun mass ratio;
 the number in parentheses is the solar mass, unit E20 kg. }
X1:=LN(X1);
X1:=X1/3.0;
X1:=EXP(X1); {Finds the cube root of X1}

D:=1.15*RV*ONEAU*X1; {Radius of the sphere of influence in metres}

CHID:=2.0*ARCTAN(GMPOU2/D);  {Minimum deflection}

END;  {of procedure CHIS}

{****************************************************************}
```

```
FUNCTION SPATIAL(A,E,I : REAL) : REAL;

{This function calculates the spatial density of an object
 at any particular point; the units are particles per cubic A.U.
 The following are the parameters:
     A   :   Semi-major axis of the orbit in AU ;
     E   :   Eccentricity      "       "         ;
     I   :   Inclination       "       "    in radians  }

VAR    Q,QD                        :   REAL;
       D1,D2,D3,D4,D5,D6           :   REAL;    {Dummy variables}
       SOFAR                       :   REAL;    {Another dummy variable}

BEGIN

Q:=A*(1.0-E);          {Perihelion distance}
QD:=A*(1.0+E);         {Aphelion   distance}
D4:=SIN(I);
D5:=SIN(BD);
D6:=SIN(B);
SOFAR:=0.0;
IF ((R <= QD) AND (RD >= Q)) THEN BEGIN   {i.e. as long as the orbit
                                             intersects the shell     }
  IF ((B <= I) AND (BD >= (-I))) THEN BEGIN  {i.e. if the volume
             element is at a latitude which the orbit can intersect}
D2:=SQR(D4);
D3:=SQR(SIN((B+BD)/2.0));
D1:=(D2-D3)*(RBAR-Q)*(QD-RBAR);
IF ((D2 <= D3) OR (RBAR <= Q) OR (QD <= RBAR)) THEN D1:=0.0
                                          ELSE D1:=SQRT(D1);
IF D1 > 0.1 THEN SOFAR:=1.0/(A*RBAR*D1*2.0*PI*PI*PI)
             ELSE BEGIN
         SOFAR:=1.0/(4.0*PI*RBAR*DELTAR);
IF ABS(QD-Q) < 0.000001 THEN BEGIN
                              D1:=1.0; D2:=-1.0;
                                END
                               ELSE BEGIN
                    D1:=((2.0*RD)-(2.0*A))/(QD-Q); LIMIT(D1);
                    D2:=((2.0*R)-(2.0*A))/(QD-Q); LIMIT(D2);
                                          END;
IF ((D1*D2) < -0.999999) THEN  {i.e. if D1=1 & D2=-1}
                SOFAR:=SOFAR/RBAR
                ELSE BEGIN
                D3:=ASIN(D1) - ASIN(D2);
                SOFAR:=SOFAR*D3/(PI*A);
                          END;
 {Now start on the latitude part}
      SOFAR:=SOFAR*2.0/(PI*(D5 - D6));
      D1:=0.0; D2:=0.0;
      SBSI(D1,D4,D5);
      SBSI(D2,D4,D6);
      D3:=ASIN(D1) - ASIN(D2);
      SOFAR:=SOFAR*D3;
                        END;   {D1 <= 0.1 loop   }
                       END;    {Latitude O.K. loop}
                      END;     {Shell intercept loop}

 SPATIAL:=SOFAR;
 END;  {of function SPATIAL}

 {*********************************************************************}
```

```
PROCEDURE VELOCITY(VAR VSIGC,VSIGD,VSIGE : REAL);

{This procedure finds the product of the mean velocity and
 the mean cross-sections in a volume element. The cross-
 sections are :        Collision  =>   VSIGC
                       Deflection =>   VSIGD
                       Ejection   =>   VSIGE                }

VAR  DUMMY1,DUMMY2,DUMMY3,V12,TWOV1V2,VA,VB,VC,VD : REAL;
     SIGMAC,SIGMAD,SIGMAE,CB,Z12                  : REAL;

{-------------------------------------------------------------}

PROCEDURE SCATTER(VAR SIGMAC,SIGMAD,SIGMAE : REAL);

{This procedure calculates the effective cross-sections against
 collision (SIGMAC), deflection (SIGMAD) and ejection (SIGMAE) }

VAR   EPS,EPSC,CHIG,CHID,D,PROB,L3,L4,L5,UC  : REAL;

{^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^}

PROCEDURE SIMPSON(VAR P : REAL);

{This procedure evaluates eqn 23 in Weidenschilling (1975a)
             using Simpson's Rule to find the integral     }

VAR  CHIMIN,CHIMAX,DCHI,CHI,A,B,C,DUMMY,SUM,FACTOR  : REAL;
     IOTA                                           : INTEGER;

{+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++}

FUNCTION WEIDEN(CHI : REAL) : REAL;

VAR X1,X2,X3 : REAL;

BEGIN
X1:=CHI/2.0;
X2:=SIN(X1);
X3:=COS(X1)/(X2*X2*X2);
X1:=(COS(EPSC) - (COS(CHI)*COS(EPS)))/(SIN(CHI)*SIN(EPS));
LIMIT(X1);
X2:=ACOS(X1);
WEIDEN:=X2*X3;
END; {of function WEIDEN}

{+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++}
```

```
BEGIN   {procedure SIMPSON}
DUMMY:=EPS-EPSC;
IF DUMMY > CHID THEN CHIMIN:=DUMMY ELSE CHIMIN:=CHID;
DUMMY:=EPS+EPSC;
IF DUMMY < CHIG THEN CHIMAX:=DUMMY ELSE CHIMAX:=CHIG;
IF ((CHIMIN < CHIG) AND (CHIMAX >CHID)) THEN BEGIN
                             {i.e. if ejection possible}
FACTOR:=((1.0/SQR(SIN(CHID/2.0))) - 1.0)*PI;
FACTOR:=1.0/FACTOR;
DCHI:=(CHIMAX-CHIMIN)/90.0;
CHI:=CHIMIN;
A:=WEIDEN(CHI);
SUM:=0.0;
FOR IOTA:=1 TO 45 DO BEGIN
    {This means that the jumps are 2 degrees or less}
  CHI:=CHI+DCHI;
  B:=WEIDEN(CHI);
  CHI:=CHI+DCHI;
  C:=WEIDEN(CHI);
  DUMMY:=A + (4.0*B) + C ;
  SUM:=SUM+DUMMY;
  A:=C;
                             END; {of IOTA loop}
SUM:=SUM*DCHI/3.0; {Complete the Simpson's Rule formula}
           P:=FACTOR*SUM;
           END   {ejection possible}
          ELSE P:=0.0; {no ejection possible}
END;   {of procedure SIMPSON}

{^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^}

BEGIN   {procedure SCATTER}
CHIS(CHIG,CHID,D,MASS1,RAD1,VREL,RBAR);
PROB:=0.0;
VRC:=SQRT(SQR(V2)+SQR(VCIRC)-(2.0*V2*VCIRC*DUMMY3));
IF (VRC>VLEAST) AND (VRC<VMOST) THEN BEGIN
  L3:=2.0*VRC*VCIRC;
  L4:=SQR(VRC);
  L5:=SQR(VCIRC);
  EPS:=ACOS((-L4-L5+SQR(V2))/L3);
  EPSC:=ACOS((-L4+L5)/L3);
  SIMPSON(PROB);
                                        END;
IF VRC>=VMOST THEN PROB:=1.0; {Ejection certain}
SIGMAD:=SQR(D)*1.0E-6/RAD1SQ;     {The cross-sections are in units
                     of the geometrical cross-section of the planet;
                     divide by a million to convert to km2           }
SIGMAE:=SIGMAD*PROB;
SIGMAC:=(1.0+(VESQ/SQR(VREL)));
   {VESQ is the square of the planetary escape velocity, in m2/s2}
END;  {of procedure SCATTER}

{------------------------------------------------------------------------}
```

```
BEGIN                {procedure VELOCITY}
VSIGC:=0.0; VSIGD:=0.0; VSIGE:=0.0;
CB:=COS((B+BD)/2.0);
SBSI(Z12,CB,COS(I1));    {Not what SBSI was written for, }
ALF1:=ACOS(Z12);         {but it'll work                }
SBSI(Z12,CB,COS(I2));
ALF2:=ACOS(Z12);

CG1:=SQRT(A1*A1*(1.0-E1)*(1.0+E1)/(RBAR*((2.0*A1)-RBAR)));
LIMIT(CG1);
SG1:=SQRT(1.0-(CG1*CG1));
CG2:=SQRT(A2*A2*(1.0-E2)*(1.0+E2)/(RBAR*((2.0*A2)-RBAR)));
LIMIT(CG2);
SG2:=SQRT(1.0-(CG2*CG2));

V1:=SQRT((RATGMONE)*((TWORBAR) - (1.0/A1)));
V2:=SQRT((RATGMONE)*((TWORBAR) - (1.0/A2)));

DUMMY1:=SG1*SG2;
DUMMY2:=CG1*CG2*COS(ALF1-ALF2);
DUMMY3:=DUMMY2/CG1;
V12:=SQR(V1)+SQR(V2); TWOV1V2:=2.0*V1*V2;
Z12:=DUMMY2+DUMMY1;
VA:=SQRT(V12-(TWOV1V2*Z12));
IF VA < VMIN THEN VMIN:=VA; IF VA > VMAX THEN VMAX:=VA;
VREL:=VA;
SCATTER(SIGMAC,SIGMAD,SIGMAE);
VSIGC:=VSIGC+(VA*SIGMAC);
VSIGD:=VSIGD+(VA*SIGMAD);
VSIGE:=VSIGE+(VA*SIGMAE);
Z12:=DUMMY2-DUMMY1;
VB:=SQRT(V12-(TWOV1V2*Z12));
IF VB < VMIN THEN VMIN:=VB; IF VB > VMAX THEN VMAX:=VB;
VREL:=VB;
SCATTER(SIGMAC,SIGMAD,SIGMAE);
VSIGC:=VSIGC+(VB*SIGMAC);
VSIGD:=VSIGD+(VB*SIGMAD);
VSIGE:=VSIGE+(VB*SIGMAE);
DUMMY2:=CG1*CG2*COS(ALF1+ALF2);
DUMMY3:=DUMMY2/CG1;
Z12:=DUMMY2+DUMMY1;
VC:=SQRT(V12-(TWOV1V2*Z12));
IF VC < VMIN THEN VMIN:=VC; IF VC > VMAX THEN VMAX:=VC;
VREL:=VC;
SCATTER(SIGMAC,SIGMAD,SIGMAE);
VSIGC:=VSIGC+(VC*SIGMAC);
VSIGD:=VSIGD+(VC*SIGMAD);
VSIGE:=VSIGE+(VC*SIGMAE);
Z12:=DUMMY2-DUMMY1;
VD:=SQRT(V12-(TWOV1V2*Z12));
IF VD < VMIN THEN VMIN:=VD; IF VD > VMAX THEN VMAX:=VD;
VREL:=VD;
SCATTER(SIGMAC,SIGMAD,SIGMAE);
VSIGC:=VSIGC+(VD*SIGMAC);
VSIGD:=VSIGD+(VD*SIGMAD);
VSIGE:=VSIGE+(VD*SIGMAE);

VSIGC:=VSIGC/4.0; VSIGD:=VSIGD/4.0; VSIGE:=VSIGE/4.0;
       {Divide by four to get the average value}
END;   {of procedure VELOCITY}

{******************************************************************}
```

```
PROCEDURE ENERGYCHANGE(VAR DELTA : REAL; CHIG,CHID : REAL);

  {DELTA is a measure of the overall energy change for an encounter}

VAR AA,VB2,V1BAR                 : REAL;
    C1,C2,C3,C4,B1,B2,B3,B4      : REAL; {Dummy variables}
BEGIN
IF (Q2<A1) AND (QD2>A1) THEN BEGIN
C1:=CHID/2.0;
C2:=SIN(C1);
B1:=1.0/SQR(C2);
B2:=2.0*COS(C1)/C2;
C3:=CHIG/2.0;
C4:=SIN(C3);
B3:=1.0/SQR(C4);
B4:=2.0*COS(C3)/C4;
CHIBAR:=((CHID*B1) + B2 - (CHIG*B3) - B4) / (B1 - B3);
C1:=8.0*LN(C2);
C3:=8.0*LN(C4);
B2:=2.0*CHID*B2;
B4:=2.0*CHIG*B4;
C1:=((SQR(CHID)*B1) - (SQR(CHIG)*B3) + B2 - B4 + C3 - C1) /(B1-B3);
CHIRMS:=SQRT(C1);
V1BAR:=SQRT(RATGMONE/A1);
VB2:=  SQRT(RATGMONE*((2.0/A1)-(1.0/A2)));
AA:=ACOS((SQR(VB2)-SQR(V1BAR)-SQR(VREL))/(2.0*V1BAR*VREL)); {epsilon}
DELTA:=(COS(AA-CHIRMS)) + (COS(AA+CHIRMS)) - (2.0*COS(AA));
DELTA:=-(DELTA*2.0*V1BAR*VREL/RATGMONE);
                                   END ELSE DELTA:=0.0;
END; {of procedure ENERGYCHANGE}

{*******************************************************************}

BEGIN      {The main program}
REWRITE(CFILE,'CDEPRINT');
{Masses below in E20 kg, radii in km, semi-major axes in AU,
 and inclinations in degrees                                    }
PSMA[1]:=0.3871; PECC[1]:=0.2056; PINC[1]:=7.004;
PRAD[1]:=2439.0;PMASS[1]:=3.302E3; {Mercury}

PSMA[2]:=0.7233; PECC[2]:=0.0068; PINC[2]:=3.393;
PRAD[2]:=6051.0;PMASS[2]:=4.870E4; {Venus}

PSMA[3]:=1.0000; PECC[3]:=0.0167; PINC[3]:=0.0;
PRAD[3]:=6378.0;PMASS[3]:=5.977E4; {Earth}

PSMA[4]:=1.524 ; PECC[4]:=0.0933; PINC[4]:=1.850;
PRAD[4]:=3397.0;PMASS[4]:=6.418E3; {Mars}

PSMA[5]:=5.203 ; PECC[5]:=0.0484; PINC[5]:=1.305;
PRAD[5]:=71398.0;PMASS[5]:=1.899E7;{Jupiter}

PSMA[6]:=9.539 ; PECC[6]:=0.0557; PINC[6]:=2.486;
PRAD[6]:=60330.0;PMASS[6]:=5.684E6;{Saturn}

PSMA[7]:=19.18 ; PECC[7]:=0.0472; PINC[7]:=0.771;
PRAD[7]:=26145.0;PMASS[7]:=8.727E5;{Uranus}

PSMA[8]:=30.06 ; PECC[8]:=0.0086; PINC[8]:=1.776;
PRAD[8]:=24700.0;PMASS[8]:=1.030E6;{Neptune}
```

```
FOR K:=1 TO 30 DO CNAME[K]:=' '; K:=0;
WRITE('What is the name of the object ? ');
WHILE NOT EOLN DO BEGIN
                  K:=K+1;
                  READ(NAMEBIT);
                  CNAME[K]:=NAMEBIT;
                   END; WRITELN;
FOR K:=1 TO 14 DO WRITELN(CFILE);
WRITE(CFILE,'           Object name:  ');
FOR K:=1 TO 30 DO WRITE(CFILE,CNAME[K]);
WRITELN(CFILE); WRITELN(CFILE);
WRITELN;WRITELN;
WRITELN('For this meteoroid............      ');
WRITE('What is the semi-major axis     ?    '); READLN(A2);
WRITE('   and the eccentricity         ?    '); READLN(E2);
WRITE('   and the inclination          ?    '); READLN(IMETEOR);
 IF (A2<0.0) THEN WRITELN('WARNING - NONSENSICAL SEMI-MAJOR AXIS');
 IF ((E2<0.0) OR (E2>1.0)) THEN WRITELN('WARNING - NON-ELLIPTIC ORBIT');
 IF ((IMETEOR<0.0) OR (IMETEOR>180.0)) THEN WRITELN('WARNING - INCLINATION IN ERROR');
WRITELN;
DUMMY:=SQRT(A2*A2*A2); {Period in years}
WRITELN(CFILE,'                        Semi-major axis = ',A2:8:2,' A.U.');
WRITELN(CFILE,'                        Eccentricity    = ',E2:10:4);
WRITELN(CFILE,'                        Inclination     = ',IMETEOR:8:2,' degrees ');
IF DUMMY < 1000.0 THEN
WRITELN(CFILE,'                        Period          = ',DUMMY:8:2,' years ') ELSE
WRITELN(CFILE,'                        Period          = ',DUMMY:8,' years ');
  WRITELN(CFILE); WRITELN(CFILE);
WRITE(CFILE,'              MERCURY    VENUS     EARTH    MARS');
WRITELN(CFILE,'     JUPITER    SATURN    URANUS    NEPTUNE ');
WRITELN(CFILE);
TCP:=0.0; TDP:=0.0; TEP:=0.0;
FOR PNO:=1 TO 8 DO BEGIN    {eight planet loop}
VMIN:=1.0E20; VMAX:=0.0; {Set up extreme values}
I2:=IMETEOR;
A1:=PSMA[PNO]; E1:=PECC[PNO]; I1:=PINC[PNO];
RAD1:=PRAD[PNO]; MASS1:=PMASS[PNO];
RAD1SQ:=SQR(RAD1);    {The square of the planetary radius}
VESQ:=2.0*6.672041E6*MASS1/RAD1; {Mass was in units of E20 kg
                  and radius in kilometres, so this is in m2/s2}
Q1:=A1*(1.0-E1); Q2:=A2*(1.0-E2);
QD1:=A1*(1.0+E1); QD2:=A2*(1.0+E2);
 IF (Q1<Q2) THEN QSMALL:=Q2 ELSE QSMALL:=Q1;
 IF (QD1<QD2) THEN QBIG:=QD1 ELSE QBIG:=QD2;
    QSMALL:=QSMALL*0.9; QBIG:=QBIG*1.1; {Make sure they overlap}
 JRANGE:=ROUND(10.0+(50.0*((QBIG-QSMALL)/QSMALL)));
 IF (JRANGE MOD 2) = 0 THEN JRANGE:=JRANGE+1; {odd number of cycles}
  {Note I1 & I2 are in degrees so far}
 IF I1>90.0 THEN I11:=180.0-I1 ELSE I11:=I1;
 IF I2>90.0 THEN I22:=180.0-I2 ELSE I22:=I2;
 IF (I11<I22) THEN BEGIN ISMALL:=-I11; IBIG:=I11; END
             ELSE BEGIN ISMALL:=-I22; IBIG:=I22; END;
 ISMALL:=ISMALL*1.1; IBIG:=IBIG*1.1;
 IF ISMALL < -90.0 THEN ISMALL:=-90.0;
 IF IBIG   >  90.0 THEN IBIG  := 90.0;
 {Go above & below the possible range of latitudes}
```

```
IF (IBIG - ISMALL) < 0.8 THEN BEGIN
                                IBIG:=0.5;
                                ISMALL:=-0.5;
                                END; {i.e. if either I is close to zero}
I1:=I1*DEGRAD; I2:=I2*DEGRAD; {Convert to radians}
KRANGE:=ROUND(IBIG-ISMALL+10.0); {jump in lat. one degree or less}
IF (KRANGE MOD 2) = 0 THEN KRANGE:=KRANGE+1; {odd number of cycles}
JUMPR:=(QBIG-QSMALL)/JRANGE; JUMPB:=(IBIG-ISMALL)/KRANGE;
DELTAR:=JUMPR;        {should be less than 0.1*R}
SUMC:=0.0; SUMD:=0.0; SUME:=0.0;
GMSUN:=6.672041*3.32958*5.9742E18;  {G*Mass of the Sun}
RATGMONE:=GMSUN/ONEAU;     {For later use}

FOR J:=1 TO JRANGE DO BEGIN    {The integration in R}
 R:=QSMALL+((J-1)*JUMPR); RD:=QSMALL+(J*JUMPR);
 RBAR:=(R+RD)/2.0;  {mean radial distance to the shell}
 TWORBAR:=2.0/RBAR; {for later use}
 VCIRC:=SQRT(TWORBAR*RATGMONE/2.0);
                {The circular velocity at RBAR A.U.'s}
 VLEAST:=(RT2-1.0)*VCIRC; {This is the minimum relative velocity
                     which would allow an ejection from the solar system}
 VMOST :=(RT2+1.0)*VCIRC; {Any reletive velocity greater than
                     this must result in an ejection }
 {VCIRC & VLEAST for use in the SCATTER procedure}
  FOR K:=1 TO KRANGE DO BEGIN   {the integration in latitude B}
   B:=ISMALL+((K-1)*JUMPB); BD:=ISMALL+(K*JUMPB);
   B:=B*DEGRAD; BD:=BD*DEGRAD; {Convert to radians}
   S1:=0.0; S2:=0.0; USIGC:=0.0;
   S1:=SPATIAL(A1,E1,I1);
   IF S1>0.0 THEN
       S2:=SPATIAL(A2,E2,I2); {Don't bother if S1 is zero}
   VOLUME:=2.0*PI*RBAR*RBAR*DELTAR*(BD-B)*COS((B+BD)/2.0);
   IF S2 > 0.0 THEN VELOCITY(USIGC,USIGD,USIGE);
                              {Don't bother if S1 or S2 is zero}
   IF USIGC > 0.0 THEN BEGIN  {Don't bother if no encounter}
      DUMMY:=S1*S2*VOLUME;
      SUMC:=SUMC+(DUMMY*USIGC);
      SUMD:=SUMD+(DUMMY*USIGD);
      SUME:=SUME+(DUMMY*USIGE);
                        END;
                     END;    {K loop}
                   END;    {J loop}
DUMMY:=ONEAU/1000.0;
DUMMY:=DUMMY*DUMMY*DUMMY;  {One cubic AU in km3}
D1:=PI*RAD1SQ/1000.0;     {Divide by 1000 since
                           velocity in metres/second}
D2:=3600.0*24.0*365.25;    {Seconds to years}
SUMC:=SUMC*D1*D2/DUMMY;
SUMD:=SUMD*D1*D2/DUMMY;
SUME:=SUME*D1*D2/DUMMY;
{The SUMs are now the collision,deflection and ejection probs/yr.}

WRITELN;
```

```
CASE PNO OF
 1:WRITELN('Mercury');
 2:WRITELN('Venus');
 3:WRITELN('Earth');
 4:WRITELN('Mars');
 5:WRITELN('Jupiter');
 6:WRITELN('Saturn');
 7:WRITELN('Uranus');
 8:WRITELN('Neptune');
END; {Case}
IF SUMC > 0.0 THEN BEGIN
     WRITELN('Collision Probability = ',SUMC,' per year ');
     WRITELN('Deflection     "       = ',SUMD,' per year ');
     WRITELN('Ejection       "       = ',SUME,' per year ');
            END  ELSE WRITELN('Cannot collide');

IF SUMC > 0.0 THEN BEGIN
     TCP:=TCP+SUMC; TDP:=TDP+SUMD; TEP:=TEP+SUME;
     VREL:=(VMIN+VMAX)/2.0;  {an average value}
     CHIS(D1,D2,DUMMY,MASS1,RAD1,VREL,A1); {D1 is CHIG, D2 is CHID}
     ENERGYCHANGE(DUMMY,D1,D2);
     DUMMY:=DUMMY*A2*SUMD; {Fractional energy change per year}
     VMIN:=VMIN/1000.0; {in km/s}
     VMAX:=VMAX/1000.0; {in km/s}
     CHIRMS:=CHIRMS/DEGRAD; CHIBAR:=CHIBAR/DEGRAD;
     POOL[PNO]:=SUMC; PDEF[PNO]:=SUMD; PEJE[PNO]:=SUME;
     MINV[PNO]:=VMIN; MAXV[PNO]:=VMAX; DKDT[PNO]:=DUMMY;
     DBAR[PNO]:=CHIBAR; DRMS[PNO]:=CHIRMS;
  END   ELSE BEGIN
        POOL[PNO]:=0.0; PDEF[PNO]:=0.0; PEJE[PNO]:=0.0;
        MINV[PNO]:=0.0; MAXV[PNO]:=0.0; DKDT[PNO]:=0.0;
        DBAR[PNO]:=0.0; DRMS[PNO]:=0.0;
               END;
                                   END;    {PNO=1 to 8}
WRITELN(CFILE,'Probability ');
WRITELN(CFILE,'(per year) of ');
WRITE(CFILE,'Collision:  ');
FOR I:=1 TO 8 DO BEGIN
  DUMMY:=POOL[I];
  IF DUMMY>0.0 THEN WRITE(CFILE,DUMMY:9,' ')
               ELSE WRITE(CFILE,'     -    ');
                   END;
WRITELN(CFILE); WRITELN(CFILE);
WRITE(CFILE,'Deflection: ');
FOR I:=1 TO 8 DO BEGIN
  DUMMY:=PDEF[I];
  IF DUMMY>0.0 THEN WRITE(CFILE,DUMMY:9,' ')
               ELSE WRITE(CFILE,'     -    ');
                   END;
WRITELN(CFILE); WRITELN(CFILE);
WRITE(CFILE,'Ejection:   ');
FOR I:=1 TO 8 DO BEGIN
  DUMMY:=PEJE[I];
  IF DUMMY>0.0 THEN WRITE(CFILE,DUMMY:9,' ')
               ELSE WRITE(CFILE,'     -    ');
                   END;
WRITELN(CFILE); WRITELN(CFILE);
```

```
WRITELN(CFILE,'Velocity(km/s)');
WRITE(CFILE,'Minimum:     ');
FOR I:=1 TO 8 DO BEGIN
  DUMMY:=MINV[I];
  IF DUMMY>0.0 THEN WRITE(CFILE,DUMMY:7:1,'    ')
              ELSE WRITE(CFILE,'      -     ');
                END;
WRITELN(CFILE); WRITELN(CFILE);
WRITE(CFILE,'Maximum:     ');
FOR I:=1 TO 8 DO BEGIN
  DUMMY:=MAXV[I];
  IF DUMMY>0.0 THEN WRITE(CFILE,DUMMY:7:1,'    ')
              ELSE WRITE(CFILE,'      -     ');
                END;
WRITELN(CFILE); WRITELN(CFILE);
WRITELN(CFILE,'Fract.Energy');
WRITE(CFILE,'Change(/yr):');
FOR I:=1 TO 8 DO BEGIN
  DUMMY:=DKDT[I];
  IF POOL[I]>0.0 THEN WRITE(CFILE,DUMMY:9,' ')
              ELSE WRITE(CFILE,'      -     ');
                END;
WRITELN(CFILE); WRITELN(CFILE);
WRITELN(CFILE,'Deflection,');
WRITE(CFILE,'Mean  (deg.):');
FOR I:=1 TO 8 DO BEGIN
  DUMMY:=DBAR[I];
  IF DUMMY>0.0 THEN
    IF DUMMY>0.002 THEN WRITE(CFILE,DUMMY:9:3,' ')
                 ELSE WRITE(CFILE,DUMMY:9,' ')
              ELSE WRITE(CFILE,'      -     ');
                END;
WRITELN(CFILE); WRITELN(CFILE);
WRITE(CFILE,'RMS   (deg.):');
FOR I:=1 TO 8 DO BEGIN
  DUMMY:=DRMS[I];
  IF DUMMY>0.0 THEN
    IF DUMMY>0.002 THEN WRITE(CFILE,DUMMY:9:3,' ')
                 ELSE WRITE(CFILE,DUMMY:9,' ')
              ELSE WRITE(CFILE,'      -     ');
                END;
WRITELN(CFILE);

WRITELN(CFILE); WRITELN(CFILE);
WRITELN(CFILE,'      ******************************************************');
WRITELN(CFILE,'      * Total Collision Probability = ',TCP:9,' per year * ');
WRITELN(CFILE,'      *   "   Deflection    "    "   = ',TDP:9,'  "    "   * ');
WRITELN(CFILE,'      *   "   Ejection      "    "   = ',TEP:9,'  "    "   * ');
WRITELN(CFILE,'      ******************************************************');
FOR K:=1 TO 4 DO WRITELN(CFILE);
CLOSE(CFILE);

END.     {of program CDE}
```

APPENDIX 2

ASTEROID COLLISIONS WITH THE TERRESTRIAL PLANETS

This appendix consists of a paper entitled 'Collisions in the solar system. I. Impacts of the Apollo-Amor-Aten asteroids upon the terrestrial planets' written in conjunction with W.J. Baggaley. It has been accepted for publication in the Monthly Notices of the Royal Astronomical Society. An extensive appendix to this paper has been wholly incorporated into chapter 2 of this thesis and is not repeated here.

This is the first of a series of papers using the techniques described in this thesis. The second paper (in preparation) will complete the analysis of asteroid collisions with the terrestrial planets by considering impacts upon Mars by asteroids other than the Apollo-Amor-Aten objects. By combing various catalogues and the monthly Minor Planet Circulars I have compiled a list of over 250 asteroids which cross the present orbit of Mars: no published up-to-date list exists. The frequency of close encounters with Mars is of immense importance regarding the supply and loss of asteroids from the main belt, and hence their appearance in eccentric planet-crossing orbits.

COLLISIONS IN THE SOLAR SYSTEM. I. IMPACTS OF THE APOLLO-AMOR-ATEN ASTEROIDS UPON THE TERRESTRIAL PLANETS

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Summary

The collision probability between each of the presently-known population of 4 Aten, 34 Apollo and 38 Amor asteroids and each of the terrestrial planets is determined by a new technique. The resulting mean collision rates, coupled with estimates of the total undiscovered population of each class, is useful in calculating the rate of removal of these bodies by the terrestrial planets, and the cratering rate on each planet by bodies of diameter in excess of 1 kilometre. The influx to the Earth is found to be one impact per 160,000 years, but this figure is biased by the inclusion of four recently-discovered low-inclination Apollos. Excluding these four the rate would be one per 250,000 years, in line with previous estimates. The impact rate is highest for the Earth, being around twice that of Venus. The rates for Mercury and Mars using the present sample are about one per 5 Myr and one per 1.5 Myr respectively.

1. *Introduction*

Over the past two decades our knowledge of the asteroids has expanded greatly. Physical studies of these bodies, reviewed by Chapman *et al* (1978), Chapman (1983) and in Gehrels (1979), have shown that distinct groups of common genesis exist. Studies of the dynamics of asteroids have shown many clusters in the orbital elements, commonly known as asteroid families. Commensurabilities, in particular with Jupiter, lead to the absence of asteroids at certain solar distances known as the Kirkwood Gaps, and these have been the subject of intense study.

Although the vast majority of asteroids inhabit the region entirely bounded by the orbits of Mars and Jupiter, there is a significant population which crosses the orbits of the terrestrial planets. Wetherill (1976) estimates that 30,000 ± 50% larger than 1 km may cross Mars' orbit, although Helin and Shoemaker (1979) give a figure of 10,000 ± 50%.

Asteroids of large semi-major axis whose paths take them closer to the sun than 1.0167 A.U. (i.e. Earth-crossers) are termed Apollo objects. A different type of Earth-crosser is an asteroid of the Aten class; these have semi-major axes $a < 1$ A.U. but aphelion distance $q' > 0.9833$ A.U. (The Earth's orbital eccentricity of 0.0167 means that it has perihelion distance 0.9833 A.U. and aphelion distance 1.0167 A.U.). Another class of asteroid, the Amor objects, are arbitrarily selected by dint of having perihelion distance $1.0167 < q < 1.3$ A.U.

Many Amor asteroids are known to have orbits which will evolve so that they become Earth-crossers, and would then be classed as Apollos (Shoemaker, Williams, Helin and Wolfe, 1979; hereafter SWHW).

There are two main reasons why the collision probability between the Apollo-Amor-Aten asteroids and each of the terrestrial planets is of interest. The first is as an estimate of cratering rates for each planet, and for the moon. This requires an estimate of the total population of each class; Helin and Shoemaker (1979) find the numbers to absolute visual magnitude V(1,0) = 18 to be:

$\sim$ 100 Atens;
700 ± 300 Apollos;
1000 - 2000 Amors;
10000 ± 5000 Mars-crossers; and
$\sim$ 5000 'Mars-grazers'.

Conversion from a visual magnitude to a physical size is severely dependent upon the assumed albedo, but these might be taken to be asteroids of diameter 1 kilometre or greater (Wetherill, 1976).

The methods of searching for planet-crossing asteroids have a large effect upon the deduced population; the above figure assumes that only of the order of one in twenty Apollos has yet been discovered. The methods for making the estimates have been discussed by Wetherill (1976) and criticized by Kresak (1978a, 1981), who finds that only about 250 Apollos should exist. There is also some disagreement between lunar and terrestrial cratering rates and the expected rate from the Apollo-Aten population (SWHW).

The second reason that the collision probability of the Apollo-Amor-Aten objects is of interest is in deducing their lifetimes and hence production rate. Although asteroid belt collisions are the most significant for the Apollos and Amors, the loss rate due to planetary collisions in the inner solar system is also appreciable. Previous estimates for their dynamical lifetimes are of the order of 2×10^7 years (Wetherill, 1967, 1976, 1979; Tedesco *et al.*, 1981). This is much shorter than the age of the solar system so that a continuous production is required. Possible sources are the asteroid belt, where close encounters might throw the colliding objects into eccentric orbits, or extinct cometary nuclei. The latter is generally favoured (SWHW; Wetherill, 1979) although this has been disputed (Levin and Simonenko, 1981; Kresak, 1981). The overall knowledge of the asteroid population has been reviewed by Hughes (1981).

A few more pertinent points can be made here. Since asteroid searches tend to be carried out close to the ecliptic plane, a bias towards low-inclination objects exists in the discovered population. These also have the highest collision probabilities with the planets, so that the overall distribution may have a lower mean collision rate than that deduced by previous researchers, and herein. This problem has been studied by Knezevic (1982).

The asteroid and comet influx to the Earth has also received much attention over the past few years as an explanation for mass extinctions of terrestrial biota Napier and Clube, 1979; Alvarez *et al*, 1980). In particular recent speculations have suggested a 26 Myr cyclicity to cometary waves entering the inner solar system (Rampino and Stothers, 1984; Whitmire and Jackson, 1984; Davis *et al*, 1984). This period is comparable to the dynamical lifetimes of the planet-crossing asteroids. If these asteroids are really extinct cometary nuclei then it is seen that possibly the present Apollo-Amor-Aten objects are not indicative of a steady-state population, but are merely the remnants of a previous cometary wave. This has been discussed in some detail by Clube and Napier (1982; 1984).

It is the aim of this paper to make an estimate, using a new method, of the Apollo-Amor-Aten collisional lifetimes, and of the impact rate with each of the terrestrial planets. The presently-discovered populations (4 Atens, 34 Apollos, 38 Amors) are taken to have orbital distributions typical of each class. The total populations are taken to be 100 Atens, 700 Apollos and 1500 Amors, following Helin and Shoemaker (1979), Wetherill and Shoemaker (1982), and Shoemaker (1983).

2. *Collisional probability calculation*

Previous techniques for evaluating collision probabilities have been derived from those of Öpik (1951, 1963, 1966). Öpik took one of the orbiting objects (i.e. the Earth) to have a circular orbit. Wetherill (1967) generalized this method to cover the case where neither orbit has zero eccentricity, and SWHW have also developed this theory. Arnold (1965a,b) used Öpik's equations in a Monte Carlo simulation, and recently Zimbelman (1984) has used numerical averaging of the equations to deduce collision probabilities between long-period comets and each of the planets.

The problem of Earth-orbiting artificial satellites led Kessler and Cour-Palais (1978) to develop a method based upon the concept of the 'spatial density' of each object (the number of objects per unit volume, on average). In an important paper Kessler (1981) outlined this method and applied it to the outer moons of Jupiter, showing that the four prograde moons had self-collisional lifetimes much shorter than the age of the solar system. Since Kessler's technique has escaped general notice, his method has been expanded and detailed in an Appendix to this paper. The method, which is analagous to the kinetic theory of gases, is extremely powerful and has wide general use . It is based upon the fact that the collision probability between two objects is given by:

$$P = \sum_j \bar{S}_{1j} \, \bar{S}_{2j} \, \bar{V}_j \, \bar{\sigma}_j \, \Delta U_j$$

where the summation is performed over all volume elements ΔU_j which are accessible to both bodies. The mean relative velocity at the position of the volume element is $\bar{V}_j$, the mean collision cross-section is $\bar{\sigma}_j$ (a function of velocity, due to gravitational focussing), and the mean spatial densities at this position are $\bar{S}_{1j}$ and $\bar{S}_{2j}$. These latter parameters are derivable from a generalization of Öpik's formulae, being dependent upon the semi-major axis (a), eccentricity (e) and inclination (i) of each of the two orbits. The argument of periapsis for each body is taken to be random: precession and secular perturbations lead to a variation in the argument of periapsis on a time-scale which is short compared to dynamical lifetimes (Öpik, 1951; SWHW). It is also assumed that the two objects do not avoid each other due to long-lived commensurabilities. Details of each step, and the method of numerical computation, are given in the Appendix.

3. *Collision probabilities for the Apollo-Amor-Aten Asteroids*

The collision probabilities with each of the terrestrial planets for the Aten, Apollo and Amor asteroids are shown in Tables 1,2 and 3 respectively. The method of calculation was as described in the Appendix. The planetary mean orbital elements were taken from the Explanatory Supplement (1974) and the physical characteristics (mass and radius) from Smith and West (1983). The asteroidal mass and radius are insignificant in this respect.

Since it is desirable to have as large a sample as possible, several recently discovered (and hence un-numbered) Apollos and Amors have been included. Most of these would become named and numbered asteroids as soon as the orbit is known sufficiently well to permit recovery at any future date. This means that the orbital elements (a,e,i) used here might be considered to be imprecise for individual asteroids. However, since all that is required is an estimate of the dynamical lifetimes of the planet-crossing bodies, the uncertainties in the orbits are of no major consequence. An exception to this comment would be a preliminary orbit which erroneously gave a very small inclination, or a perihelion/aphelion close to one of the planets, since then the calculated collision probability would be considerably enhanced. In fact, search techniques favour the discovery of such objects.

The orbital elements used, and listed in Tables 1-3, were taken variously from the TRIAD file (Bender, 1979), the Minor Planet Circulars, the Ephemerides of Minor Planets, and the Palomar-Leiden Survey (van Houten *et al*, 1970).

The four known Aten asteroids (Table 1) do not cross the orbit of Mars, although an Aten of high eccentricity ($e > 0.7$) could do in principle. Three of these bodies were listed by SWHW. They calculated impact probabilities with the Earth using the equations of Öpik (1951), and by their own derivative of that method. Three sets of results can therefore be compared:

	P_i(Öpik)	P_i(SWHW)	P_i(this paper)
	(All per 10^9 years)		
Hathor	14	14	14.7
Ra-Shalom	6.3	6.7	6.1
Aten	6.9	6.4	8.0

SWHW cautioned against the approximations used in Öpik's original derivation, and thus their own limited method. The more precise results obtained in this paper, using a method of wider general use, are seen to be in line with the previous estimates of SWHW.

Of the 34 Apollos in Table 2,5 cross the orbit of Mercury, 15 that of Venus, and all but one are Mars-crossers. It is seen that the majority of Apollos having very high collision probabilities with the Earth are recent discoveries. This is discussed in more detail later.

Especially anomalous in this respect is 1982DB, potentially the closest of the Apollos (Helin *et al*, 1984; the orbital elements given in that paper were in error). Not only does 1982 DB have a small inclination but also its perihelion distance ($q \simeq 0.95$ A.U.) makes an Earth-collision especially likely. With reference to the technique used here, as described in section 2 above and in the Appendix, this is seen to be because the mean spatial density of 1982 DB is very much enhanced at 1 A.U., and close to the ecliptic. The collisional lifetime of 1982 DB against an impact with the Earth (or Mars) is therefore only about 15 Myr; the dynamical lifetime, for severe orbital disruption by a planet or collision with belt asteroids, is of course very much shorter.

For those asteroids which were also included in the list of SWHW, in some cases updated orbits have been used here. Therefore a direct comparison of results is not possible for every asteroid, but in general comparable orbital elements render comparable collision probabilities.

From those asteroids for which the elements used here and by SWHW are basically the same, the following have been selected as examples of Earth-impact probabilities:

	P_i(Opik)	P_i(SWHW)	P_i(this paper)
		(All per 10^9 years)	
Icarus	1.6	2.0	1.8
Daedalus	1.0	1.6	1.3
Cerberus	2.5	3.1	3.1
Hermes	2.2	3.7	3.5
Midas	3.8	0.7	0.7
Tantalus	2.5	1.5	2.5

Table 3 shows the collision probabilities with Mars for 38 Amor asteroids. Since the method used in this paper utilizes osculating orbits, no account is taken of the gradual evolution of the Amor orbits due to secular perturbations. Thus only Mars of the terrestrial planets has a non-zero collision probability. In fact Earth-approaching asteroids can evolve into Earth-crossers (i.e. Amors evolve into Apollos) on a time scale of only ~ 10^4 years (Wetherill and Williams, 1968; Marsden, 1970; Wetherill and Shoemaker, 1982). About half of the Amors become Earth-crossers at some stage, the majority of these being shallow crossers (SWHW). This would imply an asteroidal perihelion close to 1 A.U., and consequently an enhanced spatial density and collision probability with the Earth, as illustrated by the note concerning 1982 XB in Table 3.

SWHW found a mean collision probability with the Earth of $\sim 1 \times 10^{-9}$ per year for a population of ~ 500 Earth-crossing Amors. This compares with their value of 2.6×10^{-9} per year for the average Apollo asteroid.

In the same way as some Amor asteroids are Earth-crossers for part of the time, the converse is true for some Apollos. These may oscillate in semi-major axis such that in some epochs $a > 1.0167$ A.U. and they would then be classified as Amors. The distinction is seen to be arbitrary and the *partition* between the three classifications in the present set is assumed to be much the same as in any other epoch. The time-variation in *number* of these objects is discussed later.

An additional point from Table 3 is that the preliminary orbits of 1982 YA and 1983 SA give these aphelion distances of 5.09 and 7.25 A.U. respectively, so that they are Jupiter-crossers. This results in a huge collision probability with Jupiter, especially so for 1982 YA since it has aphelion between the Jovian orbital extremes. Even though only 2 of the 38 Amors cross Jupiter, the mean planetary collision probability for the whole set is totally dominated by the giant planet. The lifetime against collision with Jupiter is about 10 Myr for 1982 YA and 40 Myr for 1983 SA; since a close approach resulting in gross orbital disruption is at least $\sim 10^3$ times more likely (Weidenschilling, 1975), these two objects have lifetimes against catastrophic disruption by Jupiter of only $\sim 10^4$ years. It must be reiterated that this assumes that: (i) the preliminary orbital elements are reasonably precise; (ii) precession occurs at such a rate that the argument of perihelion can be taken to be random; and (iii) the asteroids do not avoid Jupiter due to commensurabilities or other causes.

4. *Discussion*

Table 4 shows the mean collision probability (per 10^9 years) for the separate Aten/Apollo/Amor categories against each of the terrestrial planets. It is useful to compare these results against those of SWHW who calculated the Earth-Collision rates.

As indicated previously, the agreement is excellent for the Aten asteroids. Although the mean impact probability here is $\bar{P}_i = 22 \times 10^{-9}$ collisions per year (spread $\sigma_{n-1} = 26 \times 10^{-9}$, the unbiased estimate of the standard deviation of the probabilities for all four Aten asteroids) compared to 9.1×10^{-9} by SWHW, the discrepancy is mainly due to the inclusion of the low-inclination object 1954 XA. For the other three Atens the mean collision rate is 9.6×10^{-9} per year.

For the Apollos SWHW found a mean Earth-impact probability of 2.6×10^{-9} per year, whereas here the value is 5.8×10^{-9} per year (spread 11.3×10^{-9}). The major reason for the difference, and the large spread, is the inclusion here of several recently - discovered Apollos having large collision probabilities (in particular 1979 VA, 1982 DB, 1982 HR, and 1983 LC). It must be emphasized that these asteroids have orbits which are not yet well-determined, and which may have been preferentially discovered due to their low-inclination, Earth-approaching trajectories. If these 4 asteroids were rejected from the sample, the remaining 30 Apollos have a mean collision probability $\bar{P}_i \simeq 2.9 \times 10^{-9}$ per year, in good agreement with SWHW.

The only numbered asteroid to have an obviously high impact probability is (2101) Adonis which has an inclination of only 1°36. The collision probability of 11.5×10^{-9} per year compares with the values calculated by SWHW of 2.9×10^{-9} by Öpik's method and 6.3×10^{-9} by their own method. However, the eccentricity used by SWHW was different to that used here, or given in the TRIAD file (Bender, 1979).

Using the previously-stated assumed total population of each class, a net influx of asteroids larger than 1 km can now be found for each planet; the figure for Mars will only include the Apollo and Amor contributions, these being but a small fraction of all Mars-crossers.

Two of the 4 Atens and 5 of the 34 Apollos can cross Mercury, so that in total about 50 Atens and 100 Apollos can do likewise. Using the mean impact probabilities from Table 4 the net impact rate is $(2.4 \times 50 + 1.0 \times 100)$ per 10^9 years, or 1 per 4 or 5 Myr. This figure is of statistical value only. It is probably correct to within a factor of 3 or 4. No account can be taken of any hypothetical asteroids of low eccentricity and small semi-major axis since these have a low chance of discovery from the Earth. The above figure could now be used to find a cratering-rate. The crater produced by a 1 km. impacting object of course depends upon the incident velocity and local physical parameters such as the target material and acceleration due to gravity, as discussed by SWHW, and elsewhere; this facet is not covered here.

If a cometary wave were to boost the number of planet-crossing asteroids, those which crossed Jupiter would be rapidly removed and would not make a significant contribution to impacts upon the terrestrial planets. Therefore the orbital *distribution* of Apollo-Amor-Aten objects would not change appreciably from that present soon after such a wave, although the *number* would soon tail off. It has been assumed for the purposes of this paper that the present set of elements is not atypical of the orbital distribution at any other instant in time, and a simple scaling for the mean number over a long time-base could therefore be applied. SWHW find evidence from the lunar and terrestrial cratering record that the present number of Earth-crossers is larger than the mean over the past 3.3 billion years, contradicting any steady-state assumption and indicating a recent enhancement. This is in line with the suggestions of Clube and Napier (1982; 1984) who argue for periodic replenishment of the Apollo-Amor-Aten population due to large increases in the cometary influx. The Earth-impact rate found here is higher than that of SWHW, which means that the discrepency between the calculated rate and the crater record is increased. This would imply that the present population of Earth-crossing asteroids is above the long-term average; however, in view of the influence of the few low-inclination asteroids, such a deduction must be extremely tentative.

Following a similar treatment for the other planets these impact rates are derived:

Planet	Mean impact rate for all asteroids
Mercury	1 per 5×10^6 years
Venus	1 per 3×10^5 years
Earth	1 per 1.6×10^5 years
Mars	1 per 1.5×10^6 years

Note that the figure for Mercury takes no account of possible asteroids in small (a,e) orbits. The Martian impact rate is that for 700 Apollos and 1500 Amors only; the large number of Mars-crossers has not been considered but will be the subject of a later paper.

The resultant impact rate upon the Earth of asteroids larger than 1 km in diameter is therefore found to be about 6 per million years, whereas Wetherill and Shoemaker (1982) report 3 per million years. This doubling is clearly due to the influence of the recently-discovered low-inclination Apollos (1979 VA, 1982 DB, 1982 HR, and 1983 LC). Future refinement of our knowledge of the orbital parameters of these objects may decrease the collision rate found above. This might also come about if more asteroids away from the ecliptic were discovered, relieving any bias in the present sample. Clearly the collision rate of 6 per million years (reducible to 4 per million years by the rejection of the 4 Apollos mentioned above) should be considered a maximum, unless systematic, unbiased searches show that a large fraction of planet-crossing asteroids are of low inclination.

The above estimate of the Earth-impact rate used an assumed population identical to that of Wetherill and Shoemaker (1982); Kresak (1981) found that a much smaller population exists, although he finds that only ~ 20% of Apollos are extinct cometary nuclei, against the general belief. By studying recent close approaches to the Earth of asteroids and comets, Kresak (1978b,c) determined a collision rate of only 1 per 1 or 2 Myr for bodies larger than 1 km. This is an order of magnitude less than the rate deduced here. The contribution of active comets to the overall impact rate is small.

The lifetimes of the various asteroid classes against disruption by close encounters with the terrestrial planets can also be determined. For the 4 Aten asteroids, on the average they have 43 collisions per 10^9 years so that their collisional lifetime (for collisions with the terrestrial planets) is $\sim 2.5 \times 10^7$ years. Since these do not cross the asteroid belt, this is an estimate of their total collisional lifetime.

The 34 Apollo asteroids have on average 8.7 collisions per 10^9 years, and a planetary-collision lifetime of $\sim 10^8$

years. Similarly the 38 Amors have 0.29 collisions (with Mars only) per 10^9 years, and hence a collisional lifetime of $\sim 3 \times 10^{10}$ years. Of course the Apollos and Amors in general cross the asteroid belt where $\sim 6.6 \times 10^5$ large asteroids. are available for collision (Helin and Shoemaker, 1979); some can also intercept Jupiter. These two factors dominate their collisional lifetime, and will also be the subjects of future papers. Wetherill (1976) found a net lifetime of $\sim 2 \times 10^7$ years for the Apollos and $\sim 2 \times 10^8$ years for the Amors.

For the Atens, Apollos and Amors, the lifetimes above are only for *impacts* with the terrestrial planets. Large perturbations due to close but non-collisional encounters will be much more frequent, resulting in severe orbital disruption (Weidenschilling, 1975).

6. *Conclusions*

The collision probabilities between the presently-known Apollo-Amor-Aten asteroids and each of the four terrestrial planets have been used to deduce collision rates by asteroids of diameter greater than 1 km for each planet. The rate is found to be highest for the Earth, with one collision per 160,000 years. For Venus the rate is half of this, and is at least an order of magnitude less for Mercury and Mars. The total collision rate for Mars would be much higher since no account is taken of the large population of Mars-crossers. The rate for Mercury would also be higher if there were a significant undetected population of asteroids towards its orbit. These figures should be treated with caution since they are biased towards a high impact rate by the recent discovery of several Apollo asteroids of very low inclination, so that the small sample used here is probably not indicative of the entire population.

The lifetimes of these asteroids against collision with any of the terrestrial planets is $\sim 2.5\text{x}10^7$ years for the Atens, $\sim 10^8$ years for the Apollos, and $\sim 3 \times 10^{10}$ years for the Amors. For the Apollos and Amors the actual collisional lifetime would be dominated by collisions with other minor bodies or the Jovian planets; this is not so for the Atens. These figures should again be viewed with discretion due to the small, and probably biased, asteroid sample. Future discoveries of planet-crossers will allow these results to be re-assessed.

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Table 1: The Aten Asteroids

NUMBER	NAME	ELEMENTS				COLLISION PROBABILITY PER 10^9 ORBITS		
		a=(A.U.)	e =	i=(degrees)	P=(years)	MERCURY	VENUS	EARTH
2340	Hathor	0.8439	0.4498	5.86	0.775	3.23	15.4	11.4
2100	Ra-Shalom	0.8321	0.4364	15.76	0.759	0.44	5.73	4.64
2062	Aten	0.9665	0.1826	18.94	0.950	-	-	7.61
	1954XA	0.7772	0.3454	3.93	0.685	-	35.5	41.2

Table 2: The Apollo Asteroids

Number	NAME	ELEMENTS				COLLISION PROBABILITY PER 10^9 ORBITS			
		a=(A.U.)	e =	i=(degrees)	P=(years)	MERCURY	VENUS	EARTH	MARS
	1983TB	1.2715	0.8903	22.04	1.434	1.71	3.16	1.98	0.25
1566	Icarus	1.0779	0.8268	22.91	1.119	1.69	3.10	1.98	0.27
2212	Hephaistos	2.1637	0.8351	11.89	3.183	2.82	5.63	3.35	0.39
	1974MA	1.7752	0.7620	37.79	2.365	1.03	2.35	1.33	0.15
2101	Adonis	1.8749	0.7638	1.36	2.567	2.84	20.5	29.6	3.00
	1982TA	2.3030	0.7710	12.12	3.495	-	6.24	3.45	0.38
1864	Daedalus	1.4609	0.6148	22.16	1.766	-	4.20	2.24	0.25
1865	Cerberus	1.0801	0.4669	16.09	1.123	-	6.11	3.53	0.45
	Hermes (1937 UB)	1.6393	0.6236	6.22	2.099	-	15.8	7.34	0.77
1981	Midas	1.7759	0.6499	39.84	2.367	-	3.63	1.54	0.16
2201	1947XC	2.1734	0.7118	2.52	3.204	-	31.9	17.2	2.15
	1981VA	2.4600	0.7439	22.02	3.858	-	4.85	2.16	0.22
1862	Apollo	1.4712	0.5600	6.35	1.784	-	18.1	7.67	0.79
	1979XB	2.2624	0.7133	24.87	3.403	-	4.96	2.02	0.20
2063	Bacchus	1.0776	0.3495	9.42	1.119	-	26.5	7.49	0.60
	1983LC	2.6316	0.7092	1.52	4.269	-	-	32.2	3.18
1685	Toro	1.3672	0.4359	9.37	1.599	-	-	6.74[1]	0.61
2135	Aristaeus	1.5996	0.5037	23.04	2.023	-	-	2.92	0.25
	6743P-L	1.6805	0.5237	7.90	2.179	-	-	7.54	0.62
	1983VA	2.6143	0.6925	16.25	4.227	-	-	3.45	0.29
	1982HR	1.2100	0.3227	2.69	1.331	-	-	31.7	3.30
2329	Orthos	2.4042	0.6586	24.39	3.728	-	-	2.64	0.22
	1983TF2	1.3428	0.3871	7.84	1.556	-	-	9.48	0.79
1620	Geographos	1.2446	0.3355	13.32	1.389	-	-	6.24	0.87
1866	Sisyphus	1.8930	0.5394	41.15	2.605	-	-	2.51[2]	0.18
	1973NA	2.4272	0.6381	68.00	3.781	-	-	2.30	0.16
	1978CA	1.1248	0.2148	26.12	1.193	-	-	5.31	-
1863	Antinous	2.2602	0.6065	18.42	3.398	-	-	4.12	0.28
2102	Tantalus	1.2900	0.2984	64.02	1.465	-	-	3.66	0.52
	198288	1.4070	0.3548	20.94	1.669	-	-	5.16	0.35
	6344P-L	2.6186	0.6411	4.65	4.237	-	-	20.0	1.02
	198208	1.4893	0.3602	1.42	1.818	-		115.0	3.79
	1979VA	2.6354	0.6273	2.78	4.278	-	-	71.5	1.90
	1950DA	1.6834	0.5020	12.15	2.184	-	-	5.52	0.42

Notes (1) Toro has a commensurable mean motion with the Earth, precluding a collision in the present epoch (Ip and Mehra, 1973). However, close encounters with Mars will displace Toro from this resonance within about 3 x 10^6 years, so that the collision probability given is largely unaffected (Williams and Wetherill, 1973; Shoemaker et al., 1979).

(2) 1866 is a doubtful Earth-crosser (Shoemaker et al., 1979; Shoemaker 1983).

(3) The orbit of 1959LM is uncertain since it is taken from only a short arc, and so is not included here.

Table 3: The Amor Asteroids

NUMBER	NAME	ELEMENTS a=(A.U.)	e =	i=(degrees)	P=(years)	COLLISION PROBABILITY PER 10^9 ORBITS MARS
	1982XB	1.8379	0.4468	3.88	2.492	1.37[1]
2608	1978DA	2.4783	0.5866	15.64	3.901	0.329
	1980PA	1.9263	0.4586	2.16	2.674	3.22
2061	Anza	2.2647	0.5372	3.74	3.408	1.37
	1980AA	1.8915	0.4435	4.18	2.601	1.27
1917	Cuyo	2.1488	0.5048	23.99	3.150	0.258
1943	Anteros	1.4307	0.2560	8.70	1.711	0.911
1915	Quetzalcoatℓ	2.5286	0.5773	20.50	4.021	0.274
	1983RD	2.0888	0.4863	9.51	3.019	0.543
	1981QB	2.2391	0.5181	37.16	3.351	0.210
1980	Tezcatlipoca	1.7096	0.3651	26.85	2.235	0.288
1221	Amor	1.9206	0.4343	11.89	2.662	0.463
	1983RB	2.2233	0.5070	19.43	3.315	0.300
	1972RB	2.1487	0.4875	5.21	3.150	0.993
887	Alinda	2.4949	0.5578	9.25	3.941	0.546
	1982DV	2.0329	0.4571	5.93	2.899	0.885
	1982YA	3.1030	0.6409	33.22	5.466	0.208[2]
2202	Pele	2.2898	0.5124	8.79	3.465	0.588
1580	Betulia	2.1967	0.4895	52.03	3.256	0.211
1627	Ivar	1.8636	0.3969	8.44	2.544	0.661
	1982RA	1.5748	0.2838	32.98	1.976	0.326
	1977VA	1.8646	0.3939	2.97	2.546	2.03
433	Eros	1.4583	0.2229	10.83	1.761	0.826
	4788P-L	2.6117	0.5587	10.96	4.221	0.480
	1981CW	1.8779	0.3687	4.78	2.573	1.24
	1981QA	2.1515	0.4486	8.41	3.156	0.661
719	Albert	2.5839	0.5404	10.82	4.153	0.502[3]
	1983LB	2.2909	0.4786	25.40	3.467	0.282
	1983SA	4.2292	0.7147	30.78	8.697	0.230[4]
	1980YS	1.8153	0.3212	2.28	2.446	3.57
1036	Ganymed	2.6625	0.5374	26.45	4.344	0.279
2368	1977RA	2.1041	0.4132	5.26	3.052	1.12
1916	1953RA	2.2728	0.4499	12.84	3.426	0.486
2059	1963UA	2.6252	0.5259	10.99	4.253	0.528
	1982RD	2.1024	0.3946	24.99	3.048	0.344
	1982FT	1.7517	0.2777	20.07	2.318	0.449
	1979QB	2.3300	0.4423	3.38	3.557	1.98
	1980WF	2.2308	0.5141	6.41	3.332	0.788

Notes (1) The method used to determine the collision probability (finite volume elements) leads to a non-zero result for 1982 XB, which has a perihelion of 1.01673 compared to the Earth's aphelion of 1.01670. The resultant probability is 27.6 per 10^9 orbits.

(2) 1982YA crosses the orbit of Jupiter, with which is has a collision probability of 525.0 per 10^9 orbits.

(3) According to the TRIAD file (Bender, 1979), 719 Albert is a lost asteroid.

(4) 1983SA also crosses Jupiter's orbit, having a collision probability of 224.0 per 10^9 orbits.

Table 4: Mean collision probabilities ($\bar{P}_i$, per 10^9 years) and spreads (σ_{n-1}) for collisions between terrestrial planets and the Apollo-Amor-Aten asteroids.

	Atens			Apollos			Amors			All asteroids	
	n	$\bar{P}_i$	σ_{n-1}	n	$\bar{P}_i$	σ_{n-1}	n	$\bar{P}_i$	σ_{n-1}	n	$\bar{P}_i$
Mercury	2	2.4	± 2.5	5	1.0	± 0.4		-		7	1.4
Venus	3	26	± 23	15	5.4	± 6.0		-		18	8.8
Earth	4	22	± 26	34	5.8	± 11.3		-		38	7.5
Mars	0	-		33	0.41	± 0.55	38	0.29	± 0.31	71	0.35
All terrestrial planets	4	43		34	8.7		38	0.29			